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Fuzzy Inventory Model for Instantaneous Deterioration with Split Demand and Complete Backlogging

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ABSTRACT

This study develops an inventory control model for deteriorating items with a demand rate divided into two components: stock-dependent demand before inventory depletion and constant demand thereafter. The deterioration rate is assumed to be constant, and shortages are fully backlogged. To handle parameter uncertainty, all key parameters are modeled as trapezoidal fuzzy numbers and defuzzified using the signed distance method. The model determines the optimal replenishment policy by identifying the order quantity, the inventory depletion time, and the corresponding minimum total cost. Numerical examples illustrate the model in both fuzzy and crisp environments. The convexity of the total cost function is confirmed through graphical analysis, while sensitivity analysis evaluates the impact of changes and uncertainty in the fuzzy parameters on decision-making for inventory. The results demonstrate that the fuzzy model offers a robust and flexible tool for decision-making, particularly in settings where cost components and deterioration behavior are uncertain.

1. Introduction

Inventory control models play an important role in operations management by reducing costs and improving customer satisfaction in inventory management. Classical inventory models, such as the economic order quantity (EOQ) model, are typically developed under simplifying assumptions, including constant demand and the absence of item deterioration. However, these assumptions are often inadequate for representing real life scenarios, particularly in industries dealing with perishable goods such as food, pharmaceuticals, and fashion products. As a result, there has been increasing attention in the literature toward the development of more realistic inventory models that incorporate such practical complexities.

One important extension of traditional models is the consideration of deteriorating items. In practice, many products lose their utility, quality, or economic value over time, which significantly affects inventory decisions. Karimi and Mahdi [1] presented a literature review for inventory control of deteriorating items since 2018. Yadav *et al.* [2] proposed an inventory model for deteriorating items that incorporates a two-warehouse storage system. Mondal *et al.* [3] investigated a two-warehouse inventory model for deteriorating items under trade credit, incorporating partially backlogged demand.

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Puspita *et al.* [4] developed an inventory model for pharmaceutical products, where deterioration occurs over time during storage, and demand is assumed to follow a linear pattern. An inventory model for deteriorating items, in order to obtain an optimal advertising and inventory strategy, where the consumer demand rate depends on advertising effort and inventory of the items displayed in the store was developed by Dhakal *et al.* [5].

Another significant aspect of modern inventory modeling is the concept of stock-dependent demand. Unlike constant demand assumptions, stock-dependent demand reflects the phenomenon that demand is influenced by the quantity of items available for display. Higher inventory levels often attract more customers due to improved visibility and perceived abundance, thereby increasing sales. This concept has been widely studied and incorporated into various inventory models, demonstrating that demand behavior plays a crucial role in determining optimal inventory policies. Sharma *et al.* [6] formulated an inventory control model for deteriorating items in which demand is influenced by production rate, time, and inventory level. It is assumed that demand remains time-dependent throughout the period when stock is available, and subsequently becomes price-dependent once the inventory is depleted. Sharma *et al.* [7] presented a review of literature on the advances in stock dependent demand inventory for the last three decades. Two inventory models for perishable products, one with starting shortages and the other with ending shortages where demand for perishable products is dependent on price and stock was proposed by Dai *et al.* [8]. Panda *et al.* [9] analyzed a credit policy framework within a two-warehouse inventory model for deteriorating items, where demand is influenced by both price and stock levels, and shortages are partially backlogged. De *et al.* [10] developed an inventory model for non-instantaneously deteriorating items under inflationary conditions, allowing partially backlogged shortages. In their model, demand is dependent on both the price and the available stock level. Kaushik and Jitendera [11] introduced an inventory model that considers stock, price, and time factors to help retailers maximize profits by determining the optimal replenishment point and inventory cycle length. This model assumed a ramp-type demand function with a constant deterioration rate, addressing different scenarios, including stock-dependent demand, no stock-dependent demand, and no time effect.

Shortages are also an inevitable part of many real-world inventory systems. Instead of completely avoiding shortages, several studies have considered models where shortages are allowed and managed through backlogging. Sarker *et al.* [12] formulated six inventory models for products with instantaneous deterioration, focusing on the impacts of full and partial advance payment structures where demand function depends on both price and stock levels and accounts for shortages through full backlogging. Santhi and Karthikeyan [13] proposed a two-warehouse inventory model for instantaneously deteriorating items, where demand is dependent on both price and time, and shortages are assumed to be completely backlogged. Rabiun and Ali [14] optimized an inventory model for non-instantaneous deteriorating items with linear time function dependent variable demands and holding costs, shortages with backlogging. Sundararajan *et al.* [15] examined an economic order quantity (EOQ) model that incorporates the impact of inflation on a multivariable demand function for non-instantaneously deteriorating items. In this model, demand varies linearly with price and decreases exponentially over time. Additionally, shortages are permitted and are assumed to be partially backlogged. A deterministic inventory model for goods with a power demand pattern, allowing backlogged shortages was examined by San Jose *et al.* [16].

In addition to these factors, uncertainty is an inherent characteristic of most inventory systems. Parameters such as demand rate, deterioration rate, and cost components are often imprecise and cannot be accurately represented by deterministic values. To address this issue, fuzzy set theory has emerged as an effective mathematical tool for handling such vagueness and ambiguity. The representation of uncertain parameters using fuzzy numbers has enabled the development of more flexible and realistic inventory models that more accurately reflect real world conditions. De and Sana [17] studied

Table 1
Overview of related study

Authors	Year	Demand	Deterioration	Backlogging	Fuzzy Logic
Xu <i>et al.</i> [22]	2020	Time dependent	Yes	Partial	No
Khan <i>et al.</i> [23]	2020	Advertisement dependent	Yes	No Shortage	No
Singh <i>et al.</i> [24]	2022	Ramp type demand	Yes	No Shortage	Yes
Parida <i>et al.</i> [25]	2023	Time dependent	Yes	Partial	Yes
Pilati <i>et al.</i> [26]	2024	Constant	Yes	No Shortage	No
Pakhira <i>et al.</i> [27]	2024	Constant	Yes	Complete	Yes
Das <i>et al.</i> [28]	2025	Price dependent	Yes	Complete	Yes
Dave <i>et al.</i> [29]	2025	Time dependent	Yes	Partial	Yes
Moshtagh <i>et al.</i> [30]	2025	Dynamic demand	Yes	No Shortage	No
Mishra <i>et al.</i> [31]	2025	Stock dependent	No	Partial	No
This paper	2026	Stock dependent	Yes	Complete	Yes

an intuitionistic fuzzy EOQ inventory model with backlogging, utilizing score functions corresponding to both membership and non-membership functions. De [18] developed an EOQ model under fuzzy reasoning where all parameters are assumed randomized fuzzy set by means of fuzzy approximate reasoning. De *et al.* [19] explored carbon emission issues over production manufacturing system with the context of joint inventory control and sustainable trade credit financing for deteriorating items in a supplier–retailer–customer model under volumetric fuzzy system Karmakar and De [20] analyzed an EOQ model in which demand varies with time and the fluctuating number of tourists, employing fuzzy triangular norms to handle uncertainty. Sushil Kumar [21] developed a fuzzy inventory system of perishable items with reliability and time varying exponential demand.

Motivated by the above considerations, the present study develops an inventory control model that simultaneously incorporates stock-dependent demand, item deterioration, and shortages with complete backlogging within a fuzzy environment.

1.1 Novelty and Contribution

However, the proposed study introduces several novel features, which are summarized in Table 1. A review of the existing literature indicates that EOQ models under a fuzzy environment, where all parameters are treated as fuzzy, have not been adequately explored. In response to this research gap, the present study develops a fuzzy EOQ model in which all relevant parameters which includes cost components, the deterioration rate, parameters related to demand, and sensitivity of stock to consumption; are represented as trapezoidal fuzzy numbers. These fuzzy parameters are subsequently transformed into their corresponding crisp values using the signed distance method. The primary objective of the proposed model is to determine the minimum total cost, along with the optimal inventory depletion time and the optimal order quantity.

1.2 Structural outline of paper

The paper is organized as follows. All the preliminaries used in this paper are given in Section 2 followed by Section 3 which provides problem formulation of the study. Section 4 of this paper deals with the numerical computation part of the paper. Convexity of the model is shown in Section 5. Sensitivity analysis is done in the Section 6. Finally the conclusion is discussed in Section 7.

2. Preliminaries

Definition 2.1. Fuzzy Set [32]: Fuzzy set A on universe X is defined by function $\mu_A(x)$ called the membership function of set A . Membership function basically assigns a membership value to the elements from the universal set. It determines the degree of membership of each element in fuzzy set. The membership function is a real valued function which is defined as follows:- $\mu_A(x) : x \rightarrow [0, 1]$ Where,

$$\mu_A(x) = \begin{cases} 1 & x \in A \\ 0 & x \notin A \\ (0, 1) & x \text{ is partly in } A \end{cases}$$

Definition 2.2. Trapezoidal fuzzy numbers [32]: Let a_f be a trapezoidal fuzzy number which is represented as (a, b, c, d) where a, b, c, d are real numbers and $a \leq b \leq c \leq d$. Then the membership function $\mu_A(x)$ will be given as

$$\mu_A(x) = \begin{cases} 0, & x \leq a \\ \frac{x-a}{b-a}, & a < x \leq b \\ 1, & b < x \leq c \\ \frac{d-x}{d-c}, & c < x \leq d \\ 0, & x > d \end{cases}$$

Definition 2.3. α -Cut [32]: The α -cut of the fuzzy set a_f is a crisp set (denoted as A_α) and is defined as

$$A_\alpha = \{x : \mu_{a_f}(x) \geq \alpha \forall x \in X\}$$

Definition 2.4. Signed Distance Method: The signed distance defuzzification method is specifically supported by Lin and lee [33] and Yao and Wu [34]. let a_f be the trapezoidal fuzzy number where $a_f = (a, b, c, d)$. Then β_1 -cuts are given by

$$\begin{aligned} A_L(\beta_1) &= a + (b - a)\beta_1 \\ A_R(\beta_1) &= d - (d - c)\beta_1 \end{aligned}$$

The signed distance method of a trapezoidal fuzzy number $a_f = (a, b, c, d)$ can be given as

$$d(a_f, 0) = \frac{1}{2} \int_0^1 [a + (b - a)\beta_1 + d - (d - c)\beta_1] d\beta_1$$

3. Problem Formulation

In this section, the proposed inventory system is formulated mathematically. An economic order quantity (EOQ) model is developed for deteriorating items in which the demand is dependent upon stock prior to inventory depletion and becomes constant thereafter, with shortages assumed to be completely backlogged. To capture the uncertainty associated with system parameters, the model is formulated in a fuzzy environment using trapezoidal fuzzy numbers, and the optimal total cost incurred with optimal depletion time and economic order quantity is obtained.

3.1 Notations

The notations which are used for different variables are listed in Table 2

3.2 Assumptions

The assumptions for the presented mathematical model are given below:

- The proposed model assumes that, during the time period $[0, t_0)$, the demand rate varies with the available inventory and is modeled as a linear function of the inventory level, $D_0 + \beta I(t)$. Here, D_0 denotes the demand level independent of inventory availability, β measures the sensitivity of demand to inventory, and $I(t)$ represents the inventory position at time t . When the inventory level reaches zero, demand no longer depends on stock availability and remains fixed at D_0 . Any unmet demand during this phase is entirely backlogged and accumulated until the completion of the replenishment cycle at time T . Accordingly, the demand rate $D[t, I(t)]$ is specified as

$$D[t, I(t)] = \begin{cases} D_0 + \beta I(t), & 0 \leq t < t_0 \\ D_0, & t_0 \leq t < T \end{cases}$$

Table 2
List of parameters with their description and symbols

Description of variables	Symbol of the variables
Inventory level at time t	$I(t)$
Fuzzy inventory level at time t	$I(t)_f$
Ordering quantity per order	q
The quantity of products that deteriorate during the interval $[0, t_0]$	P_d
Defuzzified deterioration rate	θ
Fuzzy deterioration rate	θ_f
Fuzzy integrated market demand	$D_f[t, I(t)]$
Defuzzified integrated market demand	$D[t, I(t)]$
Defuzzified demand	D_0
Fuzzy demand rate	D_{0f}
The point of time where the inventory becomes zero	t_0
Cycle length of inventory	T
Defuzzified sensitivity of stock to consumption ($0 < \beta < 1$)	β
Fuzzy sensitivity of stock to consumption ($0 < \beta_f < 1$)	β_f
Defuzzified unit buying cost	a
Defuzzified per unit inventory holding cost per unit time	b
Defuzzified per unit shortage cost	c
Defuzzified per unit handling cost of the deteriorated product	d
Defuzzified fixed cost per order	M
Defuzzified total cost function	TC
Fuzzy unit buying cost	a_f
Fuzzy per unit inventory holding cost per unit time	b_f
Fuzzy per unit shortage cost	c_f
Fuzzy per unit handling cost of the deteriorated product	d_f
Fuzzy fixed cost per order	M_f

- The model considers a single type of deteriorating item, which starts at its maximum quality level and undergoes deterioration until time t_0 . The deterioration rate is constant i.e. θ .
- Shortages are allowed and are assumed to be fully backlogged.
- The lead time is considered negligible; that is, replenishment occurs instantaneously after placing an order.
- The length of each replenishment cycle is represented by T .
- Uncertain parameters in the model are described using fuzzy set theory. Specifically, trapezoidal fuzzy numbers are employed to represent the inherent uncertainty and imprecision present in practical inventory systems.

3.3 Mathematical Framework

At the beginning of the cycle (i.e. at $t = 0$), the inventory level attains its maximum value S_0 , which gradually decreases and reaches zero at time t_0 as a result of both demand fulfillment and deterioration. Inventory shortages are permitted after t_0 and are assumed to be fully backlogged until the end of the cycle at time T . A new replenishment occurs at $t = T$, restoring the inventory level to S_0 .

In practical decision-making contexts, parameters involved in an inventory control system cannot always be defined accurately by fixed numerical values. To overcome this limitation, the proposed model is reformulated within the framework of fuzzy set theory, where the parameters D_0 , θ and β are assumed to be uncertain in nature. For example, a decision-maker may not be able to determine a precise value of the deterioration rate; instead, information derived from market analysis may only indicate a feasible interval in which this rate is likely to occur. This ambiguity and lack of precision are addressed in the model through fuzzy set theory by representing the relevant parameters as trapezoidal fuzzy numbers, which effectively capture the uncertainty inherent in empirical data and market-based evaluations.

The demand function under the fuzzy environment, denoted by $D_f[t, I_f(t)]$, is defined as follows:

$$D_f[t, I_f(t)] = \begin{cases} D_{0f} + \beta_f I_f(t), & 0 \leq t < t_0 \\ D_{0f}, & t_0 \leq t < T \end{cases}$$

where D_{0f} , β_f and $I_f(t)$ are fuzzy parameter taken as fuzzy numbers.

Now using signed distance method to defuzzify fuzzy numbers to get crisp values. Let us assume $d(D_{0f}) = D_0$, $d(\beta_f) = \beta$ and $d(I_f(t)) = I(t)$, then defuzzified demand function would be as follows:

$$D_0[t, I(t)] = \begin{cases} D_0 + \beta I(t), & 0 \leq t < t_0 \\ D_0, & t_0 \leq t < T \end{cases}$$

The model considers items that deteriorate at a constant rate, where the deterioration rate is treated as a fuzzy parameter, denoted by θ_f , and is subsequently converted into its corresponding crisp value θ using the signed distance method.

In this model, the change in level of inventory at any point of time for a product following the demand is given in equation 1:

$$\frac{dI(t)}{dt} = \begin{cases} -D_0 - (\beta + \theta)I(t) & 0 \leq t < t_0 \\ -D_0 & t_0 \leq t < T \end{cases} \quad (1)$$

Fig. 1 represents the graphical presentation for change in inventory of this model.

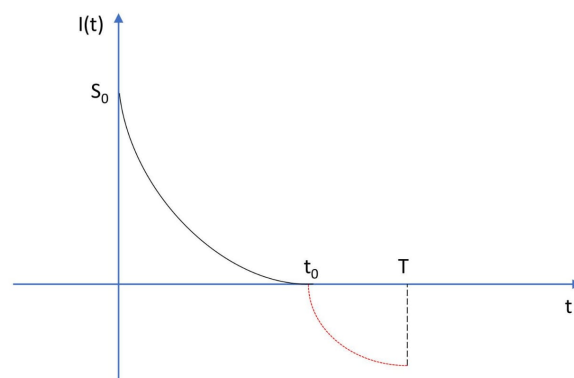


Fig. 1. Graphical representation of the inventory system

Solving equation 1 with the condition $I(t_0) = 0$ gives $I(t)$ as follows

$$I(t) = \begin{cases} \frac{D_0}{\beta+\theta} [e^{(\beta+\theta)(t_0-t)} - 1], & 0 \leq t < t_0 \\ D_0(t_0 - t), & t_0 \leq t < T \end{cases}$$

The initial inventory level can be computed by $I(0)$ in equation which gives

$$S_0 = I(0) = D_0 t_0 \left\{ 1 + \frac{(\beta + \theta)t_0}{2} \right\}$$

The total number of items deteriorated in the time interval 0 to t_0 can be computed as

$$P_d = S_0 - \int_0^{t_0} D(t, I(t))dt = \frac{D_0 t_0^2 \theta}{2} - \frac{D_0 \beta (\beta + \theta) t_0^3}{6}$$

The order quantity in this cycle is calculated as:-

$$q = S_0 + \int_{t_0}^T D(t, I(t))dt = D_0 \left\{ \frac{(\beta + \theta)t_0^2}{2} + T \right\}$$

In evaluating the total cost function, several cost components which includes purchasing cost, holding cost, storage cost, handling cost of deteriorated products, fixed ordering cost, and the deterioration rate; are modeled as fuzzy parameters. Each of these parameters is characterized by a trapezoidal fuzzy number and subsequently transformed into an equivalent crisp value through defuzzification, which enables the determination of optimal inventory policies.

Let $D_{0f} = (D_{01}, D_{02}, D_{03}, D_{04})$, $a_f = (a_1, a_2, a_3, a_4)$, $b_f = (b_1, b_2, b_3, b_4)$, $c_f = (c_1, c_2, c_3, c_4)$, $d_f = (d_1, d_2, d_3, d_4)$,

$M_f = (M_1, M_2, M_3, M_4)$ $\beta_f = (\beta_1, \beta_2, \beta_3, \beta_4)$ $\theta_f = (\theta_1, \theta_2, \theta_3, \theta_4)$ are trapezoidal fuzzy numbers.

Here, signed distance method is employed to defuzzify the fuzzy parameters. If $x_f = (x_1, x_2, x_3, x_4)$ is the fuzzy trapezoidal number then the defuzzified value of x_f using signed distance method is $x = \frac{x_1+x_2+x_3+x_4}{4}$.

Now the components of total cost function under crisp environment are computed as follows:-

Purchase cost:

$$c_1 = aq = aD_0 \left(\frac{(\beta_1 + \theta_1)t_0^2}{2} + T \right)$$

Holding cost:

$$c_2 = b \int_0^{t_0} I_1(t)dt = \frac{bD_0 t_0^2}{2} \left(1 + \frac{(\beta_1 + \theta_1)t_0}{3} \right)$$

Shortage cost:

$$c_3 = c \int_{t_0}^T (-I_1(t))dt = cD_0 \left(\frac{t_0^2}{2} + \frac{T^2}{2} - Tt_0 \right)$$

Handling cost of deteriorated products:

$$c_4 = dP_d = d \left(\frac{D_0 t_0^2 \theta_1}{2} - \frac{D_0 \beta_1 (\beta_1 + \theta_1) t_0^3}{6} \right)$$

Fixed ordering cost:

$$c_5 = M$$

The retailers total cost function is as follows:-

$$TC = aD_0 \left(\frac{(\beta_1 + \theta_1)t_0^2}{2} + T \right) + \frac{bD_0t_0^2}{+} cD_0 \left(\frac{t_0^2}{2} + \frac{T^2}{2} - Tt_0 \right) + d \left(\frac{D_0t_0^2\theta_1}{2} - \frac{D_0\beta_1(\beta_1 + \theta_1)t_0^3}{6} \right) + M \quad (2)$$

Then, this optimization problem is described as follows:

$$\text{Min } TC(t_0)$$

Subject to

$$t_0 > 0$$

The proposed model is equally applicable to crisp input values by directly replacing the fuzzy parameters with their corresponding crisp equivalents. That is, rather than using defuzzified values derived from fuzzy parameters, the model can utilize the original crisp values to determine optimal inventory decisions.

4. Numerical Computation

To demonstrate the application of the proposed model, two randomly generated datasets are considered and solved using LINGO 18.0 under both fuzzy and crisp scenarios. In Subsection 5.1, the inventory model is examined within a fuzzy framework, highlighting the optimal behavior of the total cost function when parameters are uncertain. Here, the uncertain variables are modeled as trapezoidal fuzzy numbers to reflect real-world imprecision and to evaluate the model's robustness under uncertainty. In Subsection 5.2, the same inventory model is analyzed under a crisp environment while retaining the original demand structure and model assumptions. All parameters are treated as deterministic, with exact numerical values substituted into Equation 2. The model is then solved numerically to determine the optimal time at which inventory depletes, the corresponding order quantity, and the minimum total system cost. This crisp analysis provides a computational baseline for assessing the performance of the proposed fuzzy model.

4.1 Fuzzy Model

Consider the following fuzzy parameter values:

$$\begin{aligned} T &= 60, \quad a_f = (7, 9, 11, 13), \quad b_f = (0.016, 0.018, 0.022, 0.024), \\ c_f &= (0.6, 0.8, 1.2, 1.4), \quad d_f = (2, 4, 6, 8), \quad M_f = (60, 80, 120, 140), \\ D_{0f} &= (16, 18, 22, 24), \quad \theta_f = (0.016, 0.018, 0.022, 0.024), \\ \beta_f &= (0.006, 0.008, 0.012, 0.014). \end{aligned}$$

Optimizing the above problem under fuzzy environment, we get the optimal value of defuzzified TC_f i.e. $TC_d^* = 29596.73$, optimal value for t_0 i.e. $t_0 = 26.97842$ and economic order quantity $q^* = 1333.4833$.

4.2 Crisp model

Instead of fuzzy inputs, if the crisp values are given in Equation 2 as $T = 60, a = 10, b = 0.02, c = 1, d = 5, M = 100, D_0 = 20, \theta_1 = 0.02, \beta_1 = 0.01$. Optimizing the above problem under crisp environment, we get the optimal total cost i.e. $TC^* = 35079.17$, optimal value for t_0 i.e. $t_0 = 26.04167$ and economic order quantity $q^* = 1403.4244$

A comparison of the optimal total costs from the fuzzy and crisp models reveals that the total cost under the fuzzy framework is lower than that under the crisp framework. Moreover, the inventory is depleted at a later time in the fuzzy environment.

5. Convexity of the model

5.1 Crisp Model

The convexity of the cost function TC with respect to t_0 is shown graphically in the figure 2.

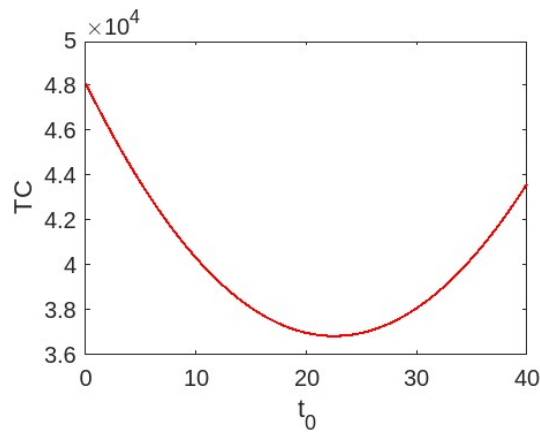


Fig. 2. Total cost as function of t_0 (crisp model)

5.2 Fuzzy Model

The convexity of the cost function TC_d with respect to t_0 is shown graphically in the figure 3.

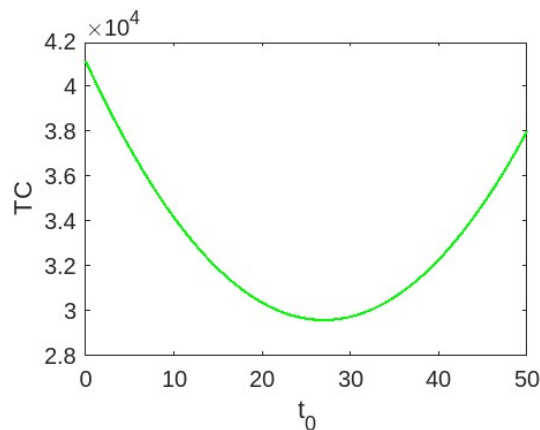


Fig. 3. Total cost as function of t_0 (fuzzy model)

6. Sensitivity Analysis

In this section, a sensitivity analysis is conducted to examine the impact of variations in key parameters on the optimal total cost of the system. Each parameter is independently decreased and increased by 50% and 25% to investigate its influence on the optimal depletion time (t_0^*), the economic order quantity (q^*), and the optimal total cost (TC_d^*) under fuzzy environments. Tables 3 to 10 provide a systematic summary of the simulated results in tabular form. The results of the study and a discussion of their implications are presented in detail in the subsequent subsections.

6.1 Results

- From Table 3 it can be observed that with increase in a_f , the optimum value of t_0 decreases while total cost increases. This suggests that when buying costs rise, inventory gets exhausted relatively faster and the retailer's overall profit declines as the model's overall total cost rises. Additionally, the model's total cost is more sensitive to changes in buying cost.
- From Table 4 it can be concluded that with increase in b_f , the optimum value of t_0 decreases while total cost increases. This means that as the inventory holding cost rises, the total profit

received by the retailer falls as the total cost of the model increases and though time t_0 has decreased but not changed much. Also, the overall cost incurred on the model is insensitive to changes in holding costs.

- From Table 5 it can be observed that with increase in c_f , the optimum value of t_0 and total cost increases. This implies that when the shortage cost increases, the inventory gets vanished later and total profit earned by the retailer decreases as the total cost for the model increases. Also, total cost incurred on the model is more sensitive towards change in shortage cost.
- From Table 6 it can be observed that with increase in d_f , the optimum value of t_0 decreases while total cost increases. This suggests that when holding cost of deteriorated products rises, inventory gets exhausted comparatively earlier and the retailer's overall profit declines as the model's overall total cost rises. Additionally, the model's total cost is less sensitive to changes in holding cost of deteriorated products.
- From Table 7 it can be concluded that with increase in M_f , the optimum value of t_0 remain same while total cost increases. This means that as the fixed cost per order rises, the total profit received by the retailer falls as the total cost of the model increases but time t_0 has not changed at all. Also, the overall cost incurred on the model is insensitive to changes in fixed ordering cost.
- From Table 8 it can be observed that with increase in D_{0f} , the optimum value of t_0 and total cost increases. This implies that when the demand increases, the inventory gets vanished at the same time and total profit earned by the retailer decreases since buying cost and holding cost for the model increases. Also, total cost incurred on the model is highly sensitive towards change in demand.
- From Table 9 it can be concluded that with increase in β_f , the optimum value of t_0 decreases while total cost increases. This means that as the sensitivity of stock to consumption rises, the total profit received by the retailer falls as the total cost of the model increases but time t_0 has not decreases much. Also, the overall cost incurred on the model is less sensitive to changes in β_f .
- From Table 10 it can be observed that with increase in θ_f , the optimum value of t_0 decreases while total cost increases. This suggests that when deterioration rate rise, inventory gets exhausted relatively earlier and the retailer's overall profit declines as the model's handling cost of deteriorated products increases leading to overall increase in total. Additionally, the model's total cost is less sensitive to changes in deterioration rate.

6.2 Discussion

According to the analysis, retailers relying on classical EOQ formulations may face unnecessarily high costs when uncertainty in demand and deterioration is overlooked. Incorporating fuzziness into the EOQ framework leads to more cost-efficient ordering policies. Moreover, fuzzy EOQ solutions enable retailers to achieve a better balance among ordering frequency, holding costs, and shortage costs, particularly for deteriorating items. Additionally, in this model, a spike in demand causes the cost of inventory to climb significantly because shortage costs would also rise, which lowers overall profit. This emphasizes how crucial it is for retailers to review methods for managing inventory as demand increases. Also, in order to get optimal profits when the the retailers need to manage sufficient inventory when the shortage cost and deterioration rate is increased.

Table 3
Sensitivity analysis with respect to parameter " a "

Change in parameter	t_0	TC_d^*	Percentage change in TC_d^*
-50 %	8.7441	32615.59	-14.9031
-25 %	8.6432	35472.29	-7.449
+25 %	8.4483	41181.82	+7.4468
+50 %	8.3540	44034.75	+14.89

Table 4
Sensitivity analysis with respect to parameter " b "

Change in parameter	t_0	TC_d^*	Percentage change in TC_d^*
-50%	27.1329	29530.66	-0.2232
-25%	27.0554	29563.79	-0.1112
+25%	26.9018	29629.48	+0.1106
+50%	26.8256	29662.04	+0.2206

Table 5
Sensitivity analysis with respect to parameter " c "

Change in parameter	t_0	TC_d^*	Percentage change in TC_d^*
-50 %	14.7783	23212.91	-21.5693
-25 %	21.82952	26902.52	-9.1030
+25 %	30.90333	31650.48	+6.9391
+50 %	33.9943	33267.88	+12.4039

Table 6
Sensitivity analysis with respect to parameter " d "

Change in parameter	t_0	TC_d^*	Percentage change in TC_d^*
-50%	29.1167	28682.57	-3.0887
-25%	28.0068	29157.07	-1.4855
+25%	26.0228	30005.23	+1.3802
+50%	25.1326	30385.79	+2.6666

Table 7
Sensitivity analysis with respect to parameter " M "

Change in parameter	t_0	TC_d^*	Percentage change in TC_d^*
-50%	26.9784	29551.73	-0.1520
-25%	26.9784	29574.23	-0.0760
+25%	26.9784	29619.23	+0.0760
+50%	26.9784	29641.73	+0.1520

Table 8
Sensitivity analysis with respect to parameter " D_0 "

Change in parameter	t_0	TC_d^*	Percentage change in TC_d^*
-50%	26.9784	14843.36	-49.8479
-25%	26.9784	22220.04	-24.9240
+25%	26.9784	36973.41	+24.9239
+50%	26.9784	44350.09	+49.8479

Table 9
Sensitivity analysis with respect to parameter " β_1 "

Change in parameter	t_0	TC_d^*	Percentage change in TC_d^*
−50%	29.0768	28699.66	-3.0309
−25%	27.9883	29164.99	-1.4587
+25%	26.0388	29998.40	+1.3571
+50%	25.1625	30373.03	+2.6229

Table 10
Sensitivity analysis with respect to parameter " θ_1 "

Change in parameter	t_0	TC_d^*	Percentage change in TC_d^*
−50%	31.8274	27523.78	-7.0039
−25%	29.2030	28645.72	-3.2132
+25%	25.0687	30413.10	+2.7583
+50%	23.4115	31121.55	+5.1519

7. Conclusions

The present study formulates an inventory control model for deteriorating items in both crisp and fuzzy settings, incorporating the assumption of fully backlogged shortages. The model is optimized to determine the minimum total cost and the optimal time at which inventory depletes. The existence of an optimal solution for the proposed model is demonstrated through numerical analysis, while its convexity is illustrated through graphical analysis. Sensitivity analysis is presented in tabular form (Tables 3 to 10) to examine the effect of variations in fuzzy parameters on the optimal defuzzified total cost. It can be observed from sensitivity analysis that optimal total cost obtained from defuzzified $\tilde{TC}$ i.e. TC_d is more sensitive to parameter $a_f, c_f, \tilde{D}_0$, less sensitive to $d_f, \tilde{\beta}_1, \tilde{\theta}_1$ and insensitive to b_f, M_f . Also the optimal total cost incurred in this model is less in fuzzy environment than crisp environment. This easily proves that fuzzy parameters provide more logical and precise results which could lead to more profit by reducing total cost spent on the inventory control model.

It may be noted that, there are further scope for enrichment by incorporating preservation technologies to alleviate the effects of item deterioration, thereby potentially reducing the associated inventory costs. In addition, extending the model to account for time-varying demand and deterioration rates would enhance its applicability to more dynamic and realistic inventory environments. Furthermore, the proposed framework may be generalized to a multi-cycle setting, allowing for a more comprehensive analysis over longer planning horizons.

Conflict of Interest

The author declares no conflicts of interest.

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