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Eco-Tourism Provider Selection using an Interval-Valued Picture Fuzzy Decision-Making Framework

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ABSTRACT

In this scientific research, uncertainty and ambiguity are handled using interval-valued picture fuzzy sets, which are particularly useful in decision-making (DM). The Hamy mean operator and Dombi operations with interval-valued picture fuzzy numbers are examined. The interval-valued picture fuzzy Dombi Hamy mean, interval-valued picture fuzzy weighted Dombi Hamy mean, interval-valued picture fuzzy dual Dombi Hamy mean, and interval-valued picture fuzzy weighted dual Dombi Hamy mean and formulated. Each Dombi Hamy aggregation operator is discussed concerning its related qualities, such as idempotency, monotonicity, bounding, and commutativity. The operators are utilized to deliver a new multiple-attribute group decision-making approach. An example of evaluating a reputable provider in the travel industry spectrum is provided.

1. Introduction

The world of things is divided into two qualities, namely membership grade (MG) and non-membership grade (NMG), in the classical set. This kind of instance only uses numerical values, for example, 0 or 1. Information, splendor, and other occurrences are just a few examples of things that the classical principle cannot adequately describe [1]. Zadeh [2] introduced the concept of fuzzy sets (FSs), which is a function that represents the idea of membership element in the interval form $[0, 1]$. This framework resolves multiple models but is unable to specify classical set theory concepts. Atanassov *et al*. [3] defined the interval-valued intuitionistic fuzzy sets (IVIFSs) based on MG and NMG. This theory increases the concept of FS. Yager *et al*. [4] developed the idea of Pythagorean fuzzy sets (PyFs) based on the MG and NMG interval of [0,1]. Yager *et al*. [5] further increased the range of MG and NG by introducing the q-rung orthopair fuzzy sets (q-ROFSs).

Cuong *et al*. [6] introduced the picture fuzzy sets (PFSs) with an additional degree, called the abstinence degree (AD). The PFS has been used by many scholars [7,8]. But sometimes the concept of PFS failed, when $0 \leq sum\left(\theta'_{Q'}(\mathbb{x}), \vartheta'_{Q'}(\mathbb{x}), \varrho'_{Q'}(\mathbb{x})\right) \leq 1$ violated. For example, the values of the

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MD, AD and NMD are 0.8, 0.2 and 0.4 respectively. In this case, $sum\left(\theta'_{Q'}(\mathbb{x}), \vartheta'_{Q'}(\mathbb{x}), \varrho'_{Q'}(\mathbb{x})\right) = 1.4 \nleq 1$. Hence, the PFS was a very limited approach [9].

Saad *et al.* [10] developed methods for multiple-attribute group decision-making (MAGDM) based on picture fuzzy Dombi Hamy mean operators. Lapo *et al.* [11] described a special type of algebras on interval-valued PFSs. Ahmad *et al*. [12] defined similarity measures based on the novel interval-valued PFSs. Kaya [13] described a two-phase group decision-making (DM) framework on the PFS-based environment. Kamacı *et al.* [14] used different aggregation operators (AOs) in the interval-valued PFS environment. Razzaque *et al.* [15] gave a multiple-attribute decision-making (MADM) approach in the PFS environment. Naeem *et al.* [16] gave the concept of the supplier selection problem in the interval-valued PFSs environment. Kumar *et al.* [17] described interval-valued PFSs in different applications. Jabeen *et al*. [18] defined the Aczel-Alsina AOs on interval-valued PFSs.

Yu & Gao [19] suggested the application of eco-tourism in a fuzzy environment. Hu & Xu [20] developed the concept of eco-agricultural tourism using the FS theory. Zhang *et al.* [21] postulated eco-tourism in the nature reserves of China. Zheng *et al.* [22] postulated eco-cultural tourism on the method of fuzzy comprehensive evaluation. Hosseini *et al.* [23] established ecotourism centers during the COVID-19 pandemic in a fuzzy environment. Fan & Xu [24] pioneered eco-tourism applications by considering a scientific calculation method.

The following is a list of this paper's contributions:

i. new AOs and some relevant attributes (such as idempotency, monotonicity and boundedness) are addressed for interval-valued PFSs;
ii. a brand-new interval-valued PFS-based MADM approach;
iii. evaluate the applicability of the introduced DM framework by addressing the eco-tourism provider selection problem.

The remainder of this research is as follows: On the basis of the Dombi operations, certain AOs for IVPFNs are developed in Section 2. In Section 3, we provide an example of how to use IVPFNs in eco-tourism provider selection problems, including sensitivity analysis. This research is concluded in Section 4.

2. Dombi Hamy Mean Operators for Interval-Valued Picture Fuzzy Numbers

The IVPFDHM operator is defined as follows by the Hamy Mean (HM) operator [25] and Dombi [26] operation rules.

Definition 1: Consider $\alpha'_{\Xi} = ([\delta_{\Xi}, \vartheta_{\Xi}], [\gamma_{\Xi}, \varepsilon_{\Xi}], [\varrho_{\Xi}, \xi_{\Xi}])(\Xi = 1, 2, \ldots, \mathbb{m})$ be a set of interval-valued picture fuzzy numbers (IVPFNs). The interval-valued picture fuzzy Dombi Hamy Mean (IVPFDHM) operator is:

$$IVPFDHM^{(\mathbb{x})}(\varphi'_1, \varphi'_2, \ldots, \varphi'_{\mathbb{m}}) = \frac{\oplus_{1 \le \mathbb{r}_1 < \cdots < \mathbb{r}_{\mathbb{x}} \le \mathbb{m}} \left(\overset{\mathbb{x}}{\underset{\Xi=1}{\otimes}} \varphi'_{\mathbb{r}_{\Xi}} \right)^{\frac{1}{\mathbb{x}}}}{\mathbb{C}_{\mathbb{m}}^{\mathbb{x}}}. \tag{1}$$

Theorem 1: Consider $\alpha'_{\Xi} = ([\delta_{\Xi}, \vartheta_{\Xi}], [\gamma_{\Xi}, \varepsilon_{\Xi}], [\varrho_{\Xi}, \xi_{\Xi}])(\Xi = 1, 2, \ldots, \mathbb{m})$ be a set of IVPFNs. The IVPFDHM operator is also an IVPFN, where:

$$IVPFDHM^{(\mathbb{x})}\left(\varphi'_1,\varphi'_2,\dots,\varphi'_{\mathbb{m}}\right)=\frac{\oplus_{1\le \mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le \mathbb{m}}\left(\bigotimes_{\varXi=1}^{\mathbb{x}}\varphi'_{\mathbb{r}_{\varXi}}\right)^{\frac{1}{\mathbb{x}}}}{\mathbb{C}_{\mathbb{m}}^{\mathbb{x}}}=$$

$$\left(\begin{matrix}\left[1-\frac{1}{1+\left[\frac{\mathbb{x}}{\mathbb{C}_{\mathbb{m}}^{\mathbb{x}}}\sum_{1\le \mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le \mathbb{m}}\frac{1}{\sum_{\varXi=1}^{\mathbb{x}}\left(\frac{1-\delta_{\mathbb{r}_{\varXi}}}{\delta_{\mathbb{r}_{\varXi}}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}},1-\frac{1}{1+\left[\frac{\mathbb{x}}{\mathbb{C}_{\mathbb{m}}^{\mathbb{x}}}\sum_{1\le \mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le \mathbb{m}}\frac{1}{\sum_{\varXi=1}^{\mathbb{x}}\left(\frac{1-\vartheta_{\mathbb{r}_{\varXi}}}{\vartheta_{\mathbb{r}_{\varXi}}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}}\right]\\ \left[\frac{1}{1+\left[\frac{\mathbb{x}}{\mathbb{C}_{\mathbb{m}}^{\mathbb{x}}}\sum_{1\le \mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le \mathbb{m}}\frac{1}{\sum_{\varXi=1}^{\mathbb{x}}\left(\frac{\gamma_{\mathbb{r}_{\varXi}}}{1-\gamma_{\mathbb{r}_{\varXi}}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}},\frac{1}{1+\left[\frac{\mathbb{x}}{\mathbb{C}_{\mathbb{m}}^{\mathbb{x}}}\sum_{1\le \mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le \mathbb{m}}\frac{1}{\sum_{\varXi=1}^{\mathbb{x}}\left(\frac{\varepsilon_{\mathbb{r}_{\varXi}}}{1-\varepsilon_{\mathbb{r}_{\varXi}}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}}\right]\\ \left[\frac{1}{1+\left[\frac{\mathbb{x}}{\mathbb{C}_{\mathbb{m}}^{\mathbb{x}}}\sum_{1\le \mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le \mathbb{m}}\frac{1}{\sum_{\varXi=1}^{\mathbb{x}}\left(\frac{\varrho_{\mathbb{r}_{\varXi}}}{1-\varrho_{\mathbb{r}_{\varXi}}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}},\frac{1}{1+\left[\frac{\mathbb{x}}{\mathbb{C}_{\mathbb{m}}^{\mathbb{x}}}\sum_{1\le \mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le \mathbb{m}}\frac{1}{\sum_{\varXi=1}^{\mathbb{x}}\left(\frac{\xi_{\mathbb{r}_{\varXi}}}{1-\xi_{\mathbb{r}_{\varXi}}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}}\right]\end{matrix}\right). \quad (2)$$

Proof of Theorem 1 is provided in Appendix 1.

Property 1 $(idempotency)$: If $\alpha'_{\varXi}=([\delta_{\varXi},\vartheta_{\varXi}],[\gamma_{\varXi},\varepsilon_{\varXi}],[\varrho_{\varXi},\xi_{\varXi}])(\varXi=1,2,\dots,\mathbb{m})=\alpha$ are equal, then:

$$IVPFDHM^{(\mathbb{x})}(\alpha'_1,\alpha'_2,\dots,\alpha'_{\mathbb{m}})=\alpha. \quad (3)$$

Property 2 $(monotonicity)$: If $\alpha'_{\varXi}=([\delta_{\varXi},\vartheta_{\varXi}],[\gamma_{\varXi},\varepsilon_{\varXi}],[\varrho_{\varXi},\xi_{\varXi}])(\varXi=1,2,\dots,\mathbb{m})$ and $\alpha'_{\varXi}=([r_{\varXi},p_{\varXi}],[m_{\varXi},l_{\varXi}],[s_{\varXi},\mathbb{m}_{\varXi}])(\varXi=1,2,\dots,\mathbb{m})$ be two sets of IVPFNs. If $\delta_{\varXi}\le r_{\varXi}$, $\vartheta_{\varXi}\le s_{\varXi}$, $\varrho_{\varXi}\le p_{\varXi}$, and $\gamma_{\varXi}\ge m_{\varXi}$, $\varepsilon_{\varXi}\ge \mathbb{m}_{\varXi}$, $\xi_{\varXi}\ge l_{\varXi}$ hold for all $\varXi$, then:

$$IVPFDHM^{(\mathbb{x})}(\alpha'_1,\alpha'_2,\dots,\alpha'_{\mathbb{m}})\le IVPFDHM^{(\mathbb{x})}(\theta'_1,\theta'_2,\dots,\theta'_{\mathbb{m}}). \quad (4)$$

Property 3 $(boundedness)$: Consider $\alpha'_{\varXi}=([\delta_{\varXi},\vartheta_{\varXi}],[\gamma_{\varXi},\varepsilon_{\varXi}],[\varrho_{\varXi},\xi_{\varXi}])(\varXi=1,2,\dots,\mathbb{m})$ be a set of IVPFNs. If ${\alpha'_{\mathbb{r}}}^{+}=([max_{\mathbb{r}}(\delta_{\varXi}),max_{\mathbb{r}}(\vartheta_{\varXi}),max_{\mathbb{r}}(\varrho_{\varXi})],[min_{\mathbb{r}}(\gamma_{\varXi}),min_{\mathbb{r}}(\varepsilon_{\varXi}),min_{\mathbb{r}}(\xi_{\varXi})])$ and if ${\alpha'_{\mathbb{r}}}^{-}=([min_{\mathbb{r}}(\delta_{\varXi}),min_{\mathbb{r}}(\vartheta_{\varXi}),min_{\mathbb{r}}(\varrho_{\varXi})],[max_{\mathbb{r}}(\gamma_{\varXi}),max_{\mathbb{r}}(\varepsilon_{\varXi}),max_{\mathbb{r}}(\xi_{\varXi})])$, then:

$$\alpha'^{-}\le IVPFDHM^{(\mathbb{x})}(\alpha'_1,\alpha'_2,\dots,\alpha'_{\mathbb{m}})\le \alpha'^{+}. \quad (5)$$

Definition 2: Consider $\alpha'_{\varXi}=([\delta_{\varXi},\vartheta_{\varXi}],[\gamma_{\varXi},\varepsilon_{\varXi}],[\varrho_{\varXi},\xi_{\varXi}])(\varXi=1,2,\dots,\mathbb{m})$ be a set of IVPFNs with their weight vector $\mathbb{w}_{\mathbb{r}}=(\mathbb{w}_1,\mathbb{w}_2,\dots,\mathbb{w}_{\mathbb{m}})^T$, thereby satisfying $\mathbb{w}_{\mathbb{r}}\in[0,1]$ and $\sum_{\mathbb{r}=1}^{\mathbb{m}}\mathbb{w}_{\mathbb{r}}=1$. Then, the interval-valued picture fuzzy weighted Dombi Hamy Mean (IVPFWDHM) operator is as follows:

$$IVPFDHM_{\mathbb{w}}^{\mathbb{x}}(\alpha'_1,\alpha'_2,\dots,\alpha'_{\mathbb{m}})=\frac{\oplus_{1\le\mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le\mathbb{m}}\left(\underset{\Xi=1}{\overset{\mathbb{x}}{\otimes}}(\alpha'_{\mathbb{r}_\Xi})^{\mathbb{w}_{\mathbb{r}_\Xi}}\right)^{\frac{1}{\mathbb{x}}}}{\mathbb{C}_{\mathbb{m}}^{\mathbb{x}}}. \tag{6}$$

Theorem 2: Consider $\alpha'_\Xi=([\delta_\Xi,\vartheta_\Xi],[\gamma_\Xi,\varepsilon_\Xi],[\varrho_\Xi,\xi_\Xi])(\Xi=1,2,\dots,\mathbb{m})(\Xi=1,2,\dots,\mathbb{m})$ be a set of IVPFNs. The IVPFWDHM operator is also an IVPFN, where:

$$IVPFDHM_{\mathbb{w}}^{\mathbb{x}}(\alpha'_1,\alpha'_2,\dots,\alpha'_{\mathbb{m}})=\frac{\oplus_{1\le\mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le\mathbb{m}}\left(\underset{\Xi=1}{\overset{\mathbb{x}}{\otimes}}(\alpha'_{\mathbb{r}_\Xi})^{\mathbb{w}_{\mathbb{r}_\Xi}}\right)^{\frac{1}{\mathbb{x}}}}{\mathbb{C}_{\mathbb{m}}^{\mathbb{x}}}=$$
$$\left(\begin{array}{c}\left[1-\frac{1}{1+\left[\frac{1}{\mathbb{C}_{\mathbb{m}}^{\mathbb{x}}}\sum_{1\le\mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le\mathbb{m}}\frac{\mathbb{x}}{\sum_{\Xi=1}^{\mathbb{x}}\mathbb{w}_{\mathbb{r}_\Xi}\left(\frac{1-\delta_{\mathbb{r}_\Xi}}{\delta_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}},1-\frac{1}{1+\left[\frac{1}{\mathbb{C}_{\mathbb{m}}^{\mathbb{x}}}\sum_{1\le\mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le\mathbb{m}}\frac{\mathbb{x}}{\sum_{\Xi=1}^{\mathbb{x}}\mathbb{w}_{\mathbb{r}_\Xi}\left(\frac{1-\vartheta_{\mathbb{r}_\Xi}}{\vartheta_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}}\right]\\ \left[\frac{1}{1+\left[\frac{1}{\mathbb{C}_{\mathbb{m}}^{\mathbb{x}}}\sum_{1\le\mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le\mathbb{m}}\frac{\mathbb{x}}{\sum_{\Xi=1}^{\mathbb{x}}\mathbb{w}_{\mathbb{r}_\Xi}\left(\frac{\gamma_{\mathbb{r}_\Xi}}{1-\gamma_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}},\frac{1}{1+\left[\frac{1}{\mathbb{C}_{\mathbb{m}}^{\mathbb{x}}}\sum_{1\le\mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le\mathbb{m}}\frac{\mathbb{x}}{\sum_{\Xi=1}^{\mathbb{x}}\mathbb{w}_{\mathbb{r}_\Xi}\left(\frac{\varepsilon_{\mathbb{r}_\Xi}}{1-\varepsilon_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}}\right]\\ \left[\frac{1}{1+\left[\frac{1}{\mathbb{C}_{\mathbb{m}}^{\mathbb{x}}}\sum_{1\le\mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le\mathbb{m}}\frac{\mathbb{x}}{\sum_{\Xi=1}^{\mathbb{x}}\mathbb{w}_{\mathbb{r}_\Xi}\left(\frac{\varrho_{\mathbb{r}_\Xi}}{1-\varrho_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}},\frac{1}{1+\left[\frac{1}{\mathbb{C}_{\mathbb{m}}^{\mathbb{x}}}\sum_{1\le\mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le\mathbb{m}}\frac{\mathbb{x}}{\sum_{\Xi=1}^{\mathbb{x}}\mathbb{w}_{\mathbb{r}_\Xi}\left(\frac{\xi_{\mathbb{r}_\Xi}}{1-\xi_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}}\right]\end{array}\right) \tag{7}$$

Definition 3: The dual DHM operator is as follows:

$$DHM^{(\mathbb{x})}(\varphi_1,\varphi_2,\dots,\varphi_{\mathbb{m}})=\left(\prod_{1\le\mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le\mathbb{m}}\left(\frac{\sum_{\Xi=1}^{\mathbb{x}}\varphi_{\mathbb{r}_\Xi}}{\mathbb{x}}\right)\right)^{\frac{1}{\mathbb{C}_{\mathbb{m}}^{\mathbb{x}}}}, \tag{8}$$

where $\mathbb{x}$ is a parameter and $\mathbb{x}=1,2,\dots,\mathbb{r}_1,\mathbb{r}_2,\dots,\mathbb{r}_{\mathbb{x}}$ are integer values taken from the set $\{1,2,\dots,\mathbb{m}\}$, $\mathbb{C}_{\mathbb{m}}^{\mathbb{x}}$ denotes the binomial coefficient, and $\mathbb{C}_{\mathbb{m}}^{\mathbb{x}}=\frac{\mathbb{m}!}{\mathbb{x}!(\mathbb{m}-\mathbb{x})!}$.

Definition 4: Consider $\alpha'_\Xi=([\delta_\Xi,\vartheta_\Xi],[\gamma_\Xi,\varepsilon_\Xi],[\varrho_\Xi,\xi_\Xi])(\Xi=1,2,\dots,\mathbb{m})$ be a set of IVPFNs. The interval-valued picture fuzzy dual Dombi Hamy Mean (IVPFDDHM) operator is:

$$IVPFDDHM^{(\mathbb{x})}(\alpha'_1,\alpha'_2,\dots,\alpha'_{\mathbb{m}})=\left(\otimes_{1\le\mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le\mathbb{m}}\left(\frac{\underset{\Xi=1}{\overset{\mathbb{x}}{\oplus}}\alpha'_{\mathbb{r}_\Xi}}{\mathbb{x}}\right)\right)^{\frac{1}{\mathbb{C}_{\mathbb{m}}^{\mathbb{x}}}}. \tag{9}$$

Theorem 3: Consider $\alpha'_\Xi=([\delta_\Xi,\vartheta_\Xi],[\gamma_\Xi,\varepsilon_\Xi],[\varrho_\Xi,\xi_\Xi])(\Xi=1,2,\dots,\mathbb{m})$ be a set of IVPFNs. The IVPFDDHM operator is also an IVPFN, where:

$$IVPFDDHM^{(\mathbb{x})}(\alpha'_1, \alpha'_2, \dots, \alpha'_{\mathbb{m}}) = \left(\otimes_{1 \le \mathbb{r}_1 < \cdots < \mathbb{r}_{\mathbb{x}} \le \mathbb{m}} \left(\frac{\overset{\mathbb{x}}{\underset{\varXi=1}{\oplus}} \alpha'_{\mathbb{r}_{\varXi}}}{\mathbb{x}} \right) \right)^{\frac{1}{\mathbb{C}_{\mathbb{m}}^{\mathbb{x}}}} =$$

$$\left(\begin{array}{c} \left[\frac{1}{1+\left[\frac{1}{\mathbb{C}_{\mathbb{m}}^{\mathbb{x}}} \sum_{1 \le \mathbb{r}_1 < \cdots < \mathbb{r}_{\mathbb{x}} \le \mathbb{m}} \frac{\mathbb{x}}{\sum_{\varXi=1}^{\mathbb{x}} \left(\frac{\delta_{\mathbb{r}_{\varXi}}}{1-\delta_{\mathbb{r}_{\varXi}}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}}, \frac{1}{1+\left[\frac{1}{\mathbb{C}_{\mathbb{m}}^{\mathbb{x}}} \sum_{1 \le \mathbb{r}_1 < \cdots < \mathbb{r}_{\mathbb{x}} \le \mathbb{m}} \frac{\mathbb{x}}{\sum_{\varXi=1}^{\mathbb{x}} \left(\frac{\vartheta_{\mathbb{r}_{\varXi}}}{1-\vartheta_{\mathbb{r}_{\varXi}}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}} \right] \\ \left[1 - \frac{1}{1+\left[\frac{1}{\mathbb{C}_{\mathbb{m}}^{\mathbb{x}}} \sum_{1 \le \mathbb{r}_1 < \cdots < \mathbb{r}_{\mathbb{x}} \le \mathbb{m}} \frac{\mathbb{x}}{\sum_{\varXi=1}^{\mathbb{x}} \left(\frac{1-\gamma_{\mathbb{r}_{\varXi}}}{\gamma_{\mathbb{r}_{\varXi}}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}}, 1 - \frac{1}{1+\left[\frac{1}{\mathbb{C}_{\mathbb{m}}^{\mathbb{x}}} \sum_{1 \le \mathbb{r}_1 < \cdots < \mathbb{r}_{\mathbb{x}} \le \mathbb{m}} \frac{\mathbb{x}}{\sum_{\varXi=1}^{\mathbb{x}} \left(\frac{1-\varepsilon_{\mathbb{r}_{\varXi}}}{\varepsilon_{\mathbb{r}_{\varXi}}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}} \right] \\ \left[1 - \frac{1}{1+\left[\frac{1}{\mathbb{C}_{\mathbb{m}}^{\mathbb{x}}} \sum_{1 \le \mathbb{r}_1 < \cdots < \mathbb{r}_{\mathbb{x}} \le \mathbb{m}} \frac{\mathbb{x}}{\sum_{\varXi=1}^{\mathbb{x}} \left(\frac{1-\varrho_{\mathbb{r}_{\varXi}}}{\varrho_{\mathbb{r}_{\varXi}}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}}, 1 - \frac{1}{1+\left[\frac{1}{\mathbb{C}_{\mathbb{m}}^{\mathbb{x}}} \sum_{1 \le \mathbb{r}_1 < \cdots < \mathbb{r}_{\mathbb{x}} \le \mathbb{m}} \frac{\mathbb{x}}{\sum_{\varXi=1}^{\mathbb{x}} \left(\frac{1-\xi_{\mathbb{r}_{\varXi}}}{\xi_{\mathbb{r}_{\varXi}}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}} \right] \end{array} \right). \tag{10}$$

Proof of Theorem 3 is provided in Appendix 2.

Property 4 ($idempotency$): If $\alpha'_{\varXi} = ([\delta_{\varXi}, \vartheta_{\varXi}], [\gamma_{\varXi}, \varepsilon_{\varXi}], [\varrho_{\varXi}, \xi_{\varXi}])(\varXi = 1, 2, \dots, \mathbb{m})$ are equal, then:

$$IVPFDDHM^{(\mathbb{x})}(\alpha'_1, \alpha'_2, \dots, \alpha'_{\mathbb{m}}) = \alpha. \tag{11}$$

Property 5 ($monotonicity$): Let $\alpha'_{\varXi} = ([\delta_{\varXi}, \vartheta_{\varXi}], [\gamma_{\varXi}, \varepsilon_{\varXi}], [\varrho_{\varXi}, \xi_{\varXi}])(\varXi = 1, 2, \dots, \mathbb{m})$ and $\alpha'_{\varXi} = ([r_{\varXi}, p_{\varXi}], [m_{\varXi}, l_{\varXi}], [s_{\varXi}, \mathbb{m}_{\varXi}])(\varXi = 1, 2, \dots, \mathbb{m})$ be two sets of IVPFNs. If $\delta_{\varXi} \le r_{\varXi}$, $\vartheta_{\varXi} \le s_{\varXi}$, $\varrho_{\varXi} \le p_{\varXi}$, and $\gamma_{\varXi} \ge m_{\varXi}$, $\varepsilon_{\varXi} \ge \mathbb{m}_{\varXi}$, $\xi_{\varXi} \ge l_{\varXi}$ hold for all $\varXi$, then:

$$IVPFDDHM^{(\mathbb{x})}(\alpha'_1, \alpha'_2, \dots, \alpha'_{\mathbb{m}}) \le IVPFDDHM^{(\mathbb{x})}(\theta'_1, \theta'_2, \dots, \theta'_{\mathbb{m}}). \tag{12}$$

Property 6 ($boundedness$): Consider $\alpha'_{\varXi} = ([\delta_{\varXi}, \vartheta_{\varXi}], [\gamma_{\varXi}, \varepsilon_{\varXi}], [\varrho_{\varXi}, \xi_{\varXi}])(\varXi = 1, 2, \dots, \mathbb{m})$ be a set of IVPFNs. If ${\alpha'_{\mathbb{r}}}^{+} = ([max_{\mathbb{r}}(\delta_{\varXi}), max_{\mathbb{r}}(\vartheta_{\varXi}), max_{\mathbb{r}}(\varrho_{\varXi})], [min_{\mathbb{r}}(\gamma_{\varXi}), min_{\mathbb{r}}(\varepsilon_{\varXi}), min_{\mathbb{r}}(\xi_{\varXi})])$ and if ${\alpha'_{\mathbb{r}}}^{-} = ([min_{\mathbb{r}}(\delta_{\varXi}), min_{\mathbb{r}}(\vartheta_{\varXi}), min_{\mathbb{r}}(\varrho_{\varXi})], [max_{\mathbb{r}}(\gamma_{\varXi}), max_{\mathbb{r}}(\varepsilon_{\varXi}), max_{\mathbb{r}}(\xi_{\varXi})])$, then:

$$\alpha'^{-} \le IVPFDDHM^{(\mathbb{x})}(\alpha'_1, \alpha'_2, \dots, \alpha'_{\mathbb{m}}) \le \alpha'^{+}. \tag{13}$$

Definition 5: Consider $\alpha'_{\varXi} = ([\delta_{\varXi}, \vartheta_{\varXi}], [\gamma_{\varXi}, \varepsilon_{\varXi}], [\varrho_{\varXi}, \xi_{\varXi}])$ $(\varXi = 1, 2, \dots, \mathbb{m})$ be a set of IVPFNs with their weight vector $\mathbb{w}_{\mathbb{r}} = (\mathbb{w}_1, \mathbb{w}_2, \dots, \mathbb{w}_{\mathbb{m}})^T$, thereby satisfying $\mathbb{w}_{\mathbb{r}} \in [0, 1]$ and $\sum_{\mathbb{r}=1}^{\mathbb{m}} \mathbb{w}_{\mathbb{r}} = 1$. Then, the interval-valued picture fuzzy weighted dual Dombi Hamy Mean (IVPFWDDHM) operator is as follows:

$$IVPFDDHM_{\mathrm{w}}^{\mathrm{x}}(\varphi'_1, \varphi'_2, \ldots, \varphi'_{\mathrm{m}}) = \left(\otimes_{1 \leq \mathrm{r}_1 < \cdots < \mathrm{r}_{\mathrm{x}} \leq \mathrm{m}} \left(\frac{\oplus_{\Xi=1}^{\mathrm{x}} \mathrm{w}_{\mathrm{r}_\Xi} \varphi'_{\mathrm{r}_\Xi}}{\mathrm{x}} \right) \right)^{\frac{1}{\mathbb{C}_{\mathrm{m}}^{\mathrm{x}}}}. \quad (14)$$

Theorem 4: Consider $\alpha'_{\Xi} = ([\delta_{\Xi}, \vartheta_{\Xi}], [\gamma_{\Xi}, \varepsilon_{\Xi}], [\varrho_{\Xi}, \xi_{\Xi}])(\Xi = 1, 2, \ldots, \mathrm{m})$ be a set of IVPFNs. The IVPFWDDHM operator is also an IVPFN, where:

$$IVPFDDHM_{\mathrm{w}}^{\mathrm{x}}(\varphi'_1, \varphi'_2, \ldots, \varphi'_{\mathrm{m}}) = \left(\otimes_{1 \leq \mathrm{r}_1 < \cdots < \mathrm{r}_{\mathrm{x}} \leq \mathrm{m}} \left(\frac{\oplus_{\Xi=1}^{\mathrm{x}} \mathrm{w}_{\mathrm{r}_\Xi} \varphi'_{\mathrm{r}_\Xi}}{\mathrm{x}} \right) \right)^{\frac{1}{\mathbb{C}_{\mathrm{m}}^{\mathrm{x}}}} =$$

$$\left(\begin{array}{c} \left[\frac{1}{1+\left[\frac{1}{\mathbb{C}_{\mathrm{m}}^{\mathrm{x}}} \sum_{1 \leq \mathrm{r}_1 < \cdots < \mathrm{r}_{\mathrm{x}} \leq \mathrm{m}} \frac{\mathrm{x}}{\sum_{\Xi=1}^{\mathrm{x}} \mathrm{w}_{\mathrm{r}_\Xi} \left(\frac{\delta_{\mathrm{r}_\Xi}}{1-\delta_{\mathrm{r}_\Xi}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}}, \frac{1}{1+\left[\frac{1}{\mathbb{C}_{\mathrm{m}}^{\mathrm{x}}} \sum_{1 \leq \mathrm{r}_1 < \cdots < \mathrm{r}_{\mathrm{x}} \leq \mathrm{m}} \frac{\mathrm{x}}{\sum_{\Xi=1}^{\mathrm{x}} \mathrm{w}_{\mathrm{r}_\Xi} \left(\frac{\vartheta_{\mathrm{r}_\Xi}}{1-\vartheta_{\mathrm{r}_\Xi}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}} \right] \\ \left[1 - \frac{1}{1+\left[\frac{1}{\mathbb{C}_{\mathrm{m}}^{\mathrm{x}}} \sum_{1 \leq \mathrm{r}_1 < \cdots < \mathrm{r}_{\mathrm{x}} \leq \mathrm{m}} \frac{\mathrm{x}}{\sum_{\Xi=1}^{\mathrm{x}} \mathrm{w}_{\mathrm{r}_\Xi} \left(\frac{1-\gamma_{\mathrm{r}_\Xi}}{\gamma_{\mathrm{r}_\Xi}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}}, 1 - \frac{1}{1+\left[\frac{1}{\mathbb{C}_{\mathrm{m}}^{\mathrm{x}}} \sum_{1 \leq \mathrm{r}_1 < \cdots < \mathrm{r}_{\mathrm{x}} \leq \mathrm{m}} \frac{\mathrm{x}}{\sum_{\Xi=1}^{\mathrm{x}} \mathrm{w}_{\mathrm{r}_\Xi} \left(\frac{1-\varepsilon_{\mathrm{r}_\Xi}}{\varepsilon_{\mathrm{r}_\Xi}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}} \right] \\ \left[1 - \frac{1}{1+\left[\frac{1}{\mathbb{C}_{\mathrm{m}}^{\mathrm{x}}} \sum_{1 \leq \mathrm{r}_1 < \cdots < \mathrm{r}_{\mathrm{x}} \leq \mathrm{m}} \frac{\mathrm{x}}{\sum_{\Xi=1}^{\mathrm{x}} \mathrm{w}_{\mathrm{r}_\Xi} \left(\frac{1-\varrho_{\mathrm{r}_\Xi}}{\varrho_{\mathrm{r}_\Xi}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}}, 1 - \frac{1}{1+\left[\frac{1}{\mathbb{C}_{\mathrm{m}}^{\mathrm{x}}} \sum_{1 \leq \mathrm{r}_1 < \cdots < \mathrm{r}_{\mathrm{x}} \leq \mathrm{m}} \frac{\mathrm{x}}{\sum_{\Xi=1}^{\mathrm{x}} \mathrm{w}_{\mathrm{r}_\Xi} \left(\frac{1-\xi_{\mathrm{r}_\Xi}}{\xi_{\mathrm{r}_\Xi}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}} \right] \end{array} \right). \quad (15)$$

3. Eco-Tourism Provider Selection

Eco-tourism has become increasingly popular around the world as living standards have increased and people's interest in environmental sustainability has grown. To draw in eco-aware tourists, many areas are concentrating on creating eco-tourism destinations. Local governments and tourism planners are finding it more and more crucial to assess the development potential and service quality of eco-tourism destinations as this industry grows. However, because there are several competing criteria, evaluating such destinations frequently involves MADM problems. We provide an example to alleviate the ambiguity in evaluation resulting from MADM in the context of IVPFNs.

Assume that five ecotourism locations require evaluation. Experts evaluate these options according to four criteria:

i. conservation efforts for the environment (E_1);
ii. accessibility and transportation facilities (E_2);
iii. participation of the local community (E_3);

iv. cost effectiveness and economic sustainability (E_4).

IVPFNs are used to express the five alternatives' assessments of these attributes (Table 1), and the attribute weights are provided as $\mathbb{w} = (0.4, 0.1, 0.3, 0.2)$.

Table 1
IVPFN decision matrix

	E_1	E_2
B_1	([0.2,0.4],[0.1,0.3],[0.2,0.3])	([0.2,0.3],[0.2,0.3],[0.1,0.2])
B_2	([0.2,0.3],[0.1,0.2],[0.1,0.5])	([0.1,0.4],[0.1,0.2],[0.2,0.4])
B_3	([0.3,0.4],[0.2,0.3],[0.1,0.3])	([0.1,0.2],[0.2,0.3],[0.3,0.4])
B_4	([0.2,0.3],[0.2,0.3],[0.1,0.3])	([0.2,0.3],[0.1,0.4],[0.2,0.3])
B_5	([0.3,0.4],[0.1,0.2],[0.3,0.4])	([0.1,0.4],[0.2,0.4],[0.1,0.2])
	E_3	E_4
B_1	([0.1,0.3],[0.1,0.2],[0.2,0.3])	([0.2,0.3],[0.1,0.2],[0.3,0.4])
B_2	([0.2,0.4],[0.1,0.3],[0.1,0.3])	([0.1,0.4],[0.2,0.4],[0.1,0.2])
B_3	([0.1,0.3],[0.2,0.4],[0.1,0.3])	([0.2,0.3],[0.1,0.2],[0.1,0.2])
B_4	([0.2,0.3],[0.1,0.3],[0.3,0.4])	([0.2,0.3],[0.2,0.3],[0.1,0.2])
B_5	([0.1,0.3],[0.1,0.2],[0.2,0.4])	([0.1,0.2],[0.1,0.2],[0.2,0.3])

Step 1: We fuse all IVPFNs $r_{\mathbb{r}\Xi}$ by the IVIFWDHM (IVPFWDDHM) operator to have the IVPFNs $B_{\mathbb{r}}(\mathbb{r} = 1, 2, \ldots, 5)$. Consider $\mathbb{x} = 2$. Then, the fused results are in Table 2.

Table 2
Results by IVPFWDHM and IVPFWDDH

	IVPFWDHM
B_1	([0.2560,0.2360],[0.1613,0.1659],[0.2638,0.2316])
B_2	([0.2133,0.3246],[0.1729,0.1873],[0.1613,0.1999])
B_3	([0.2388,0.2238],[0.2500,0.2233],[0.1622,0.2207])
B_4	([0.2892,0.2370],[0.2132,0.2253],[0.2550,0.2077])
B_5	([0.5887,0.3909],[0.5853,0.3909],[0.5903,0.3963])
	IVPFWDDHM
B_1	([0.9168,0.5131],[0.9296,0.5702],[0.9330,0.6083])
B_2	([0.9112,0.5045],[0.9231,0.6355],[0.9231,0.5527])
B_3	([0.9040,0.5949],[0.8827,0.5939],[0.9172,0.6287])
B_4	([0.8668,0.5891],[0.9074,0.5775],[0.9022,0.6089])
B_5	([0.9257,0.5735],[0.9233,0.6972],[0.8619,0.5977])

Step 2: Using Table 2, the score values are given in Table 3.

Table 3
Score values of eco-tourism options

Alternatives	IVPFWDHM	IVPFWDDHM
B_1	0.0258	0.3626
B_2	-0.0548	0.3550
B_3	0.0004	0.2955
B_4	0.0291	0.3003
B_5	-0.0088	0.2808

Step 3: Table 4 lists the tourist attractions' scenic locations from Table 3. Moreover, the ideal tourist destination is B_1.

Table 4
Order of the tourism scenic location alternatives

Methods	Order
IVPFWDHM	$B_1>B_4>B_3>B_5>B_2$
IVPFWDDHM	$B_1>B_2>B_4>B_3>B_5$

3.1 Sensitivity Analysis

The sensitivity findings are included in Tables 5 and Table 6 to show how changing the parameters for the IVIFWDHM and IVIFWDDHM operators affects the ordering.

Table 5
Sensitivity analysis results regarding the IVPFWDHM operator

	$S(B_1)$	$S(B_2)$	$S(B_3)$	$S(B_4)$	$S(B_5)$	Order
$X=1$	0.0167	−0.0860	−0.0320	0.0055	−0.0404	$B_5>B_4>B_1>B_3>B_2$
$X=2$	0.0258	−0.0548	0.0004	0.0291	−0.0088	$B_1>B_2>B_3>B_5>B_2$
$X=3$	0.1989	0.0206	0.0784	0.1124	0.0673	$B_1>B_4>B_3>B_5>B_2$
$X=4$	0.3963	0.2164	0.2782	0.3232	0.2628	$B_1>B_4>B_3>B_5>B_2$

Table 6
Sensitivity analysis results regarding the IVPFWDDHM operator

	$S(B_1)$	$S(B_2)$	$S(B_3)$	$S(B_4)$	$S(B_5)$	Order
$X=1$	0.3325	0.3244	0.2640	0.2678	0.2503	$B_1>B_2>B_4>B_3>B_5$
$X=2$	0.3626	0.3550	0.2955	0.3003	0.2808	$B_1>B_2>B_4>B_3>B_5$
$X=3$	0.4339	0.4272	0.3704	0.3773	0.3539	$B_1>B_2>B_4>B_4>B_5$
$X=4$	0.6044	0.5994	0.5524	0.5618	0.5336	$B_1>B_2>B_4>B_3>B_5$

4. Conclusions

In this article, we examined the MADM issues with IVPFNs. Next, we developed some AOs using IVPFNs using the HM operator and Dombi operations; i.e., IVPFDHM operator, IVPFWDHM operator, IVPFDDHM operator, and IVPFWDDHM operator. Then, we suggested two approaches addressing MADM issues with IVPFNs using the IVPFWDHM and IVPFWDDHM operators. Finally, a real-world example of evaluating an outdated eco-tourism service's quality within an eco-tourist destination is utilized to illustrate the methodology.

In future research endeavors, the extension and application of IVPFNs must be investigated in a variety of diverse uncertain settings and distinct packages in the ensuing investigations. The proposed operator will therefore be developed in the context of interval-valued spherical FSs and T-spherical FSs. Extending the developed AOs in complex theory is another goal.

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Conflicts of Interest
The author declares no conflicts of interest.

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Appendix 1: Proof of Theorem 1

$$\bigotimes_{\Xi=1}^{\mathbb{x}} \alpha'_{\mathbb{r}_\Xi} = \left(\begin{array}{c} \left[\frac{1}{1+\left(\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{1-\delta_{\mathbb{r}_\Xi}}{\delta_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}\right)^{\frac{1}{\mathcal{L}}}}, \frac{1}{1+\left(\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{1-\vartheta_{\mathbb{r}_\Xi}}{\vartheta_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}\right)^{\frac{1}{\mathcal{L}}}} \right] \\ \left[1-\frac{1}{1+\left(\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{\gamma_{\mathbb{r}_\Xi}}{1-\gamma_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}\right)^{\frac{1}{\mathcal{L}}}}, 1-\frac{1}{1+\left(\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{\varepsilon_{\mathbb{r}_\Xi}}{1-\varepsilon_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}\right)^{\frac{1}{\mathcal{L}}}} \right] \\ \left[1-\frac{1}{1+\left(\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{\varrho_{\mathbb{r}_\Xi}}{1-\varrho_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}\right)^{\frac{1}{\mathcal{L}}}}, 1-\frac{1}{1+\left(\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{\xi_{\mathbb{r}_\Xi}}{1-\xi_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}\right)^{\frac{1}{\mathcal{L}}}} \right] \end{array} \right),$$

$$\left(\bigotimes_{\Xi=1}^{\mathbb{x}} \alpha'_{\mathbb{r}_\Xi}\right)^{\frac{1}{\mathbb{x}}} = \left(\begin{array}{c} \left[\frac{1}{1+\left(\frac{1}{\mathbb{x}}\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{1-\delta_{\mathbb{r}_\Xi}}{\delta_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}\right)^{\frac{1}{\mathcal{L}}}}, \frac{1}{1+\left(\frac{1}{\mathbb{x}}\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{1-\vartheta_{\mathbb{r}_\Xi}}{\vartheta_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}\right)^{\frac{1}{\mathcal{L}}}} \right] \\ \left[1-\frac{1}{1+\left(\frac{1}{\mathbb{x}}\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{\gamma_{\mathbb{r}_\Xi}}{1-\gamma_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}\right)^{\frac{1}{\mathcal{L}}}}, 1-\frac{1}{1+\left(\frac{1}{\mathbb{x}}\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{\varepsilon_{\mathbb{r}_\Xi}}{1-\varepsilon_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}\right)^{\frac{1}{\mathcal{L}}}} \right] \\ \left[1-\frac{1}{1+\left(\frac{1}{\mathbb{x}}\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{\varrho_{\mathbb{r}_\Xi}}{1-\varrho_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}\right)^{\frac{1}{\mathcal{L}}}}, 1-\frac{1}{1+\left(\frac{1}{\mathbb{x}}\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{\xi_{\mathbb{r}_\Xi}}{1-\xi_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}\right)^{\frac{1}{\mathcal{L}}}} \right] \end{array} \right),$$

$$\bigoplus_{1\le \mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le \mathbb{m}} \left(\bigotimes_{\Xi=1}^{\mathbb{x}} \alpha'_{\mathbb{r}_\Xi}\right)^{\frac{1}{\mathbb{x}}} = \left(\begin{array}{c} \left[1-\frac{1}{1+\left[\sum_{1\le \mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le \mathbb{m}} \frac{\mathbb{x}}{\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{1-\delta_{\mathbb{r}_\Xi}}{\delta_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}}, 1-\frac{1}{1+\left[\sum_{1\le \mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le \mathbb{m}} \frac{\mathbb{x}}{\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{1-\vartheta_{\mathbb{r}_\Xi}}{\vartheta_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}} \right] \\ \left[\frac{1}{1+\left[\sum_{1\le \mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le \mathbb{m}} \frac{\mathbb{x}}{\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{\gamma_{\mathbb{r}_\Xi}}{1-\gamma_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}}, \frac{1}{1+\left[\sum_{1\le \mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le \mathbb{m}} \frac{\mathbb{x}}{\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{\varepsilon_{\mathbb{r}_\Xi}}{1-\varepsilon_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}} \right] \\ \left[\frac{1}{1+\left[\sum_{1\le \mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le \mathbb{m}} \frac{\mathbb{x}}{\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{\varrho_{\mathbb{r}_\Xi}}{1-\varrho_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}}, \frac{1}{1+\left[\sum_{1\le \mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le \mathbb{m}} \frac{\mathbb{x}}{\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{\xi_{\mathbb{r}_\Xi}}{1-\xi_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}} \right] \end{array} \right).$$

Therefore, we have:

$$IVPFDHM^{(\mathbb{x})}(\alpha'_1, \alpha'_2, \dots, \alpha'_{\mathbb{m}}) = \frac{\oplus_{1\le \mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le \mathbb{m}}\left(\overset{\mathbb{x}}{\underset{\Xi=1}{\otimes}} \alpha'_{\mathbb{r}_\Xi}\right)^{\frac{1}{\mathbb{x}}}}{\mathbb{C}_{\mathbb{m}}^{\mathbb{x}}} =$$

$$\left(\begin{array}{c} \left[1-\frac{1}{1+\left[\frac{\mathbb{x}}{\mathbb{C}_{\mathbb{m}}^{\mathbb{x}}}\sum_{1\le \mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le \mathbb{m}}\frac{1}{\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{1-\delta_{\mathbb{r}_\Xi}}{\delta_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}}, 1-\frac{1}{1+\left[\frac{\mathbb{x}}{\mathbb{C}_{\mathbb{m}}^{\mathbb{x}}}\sum_{1\le \mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le \mathbb{m}}\frac{1}{\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{1-\vartheta_{\mathbb{r}_\Xi}}{\vartheta_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}}\right] \\ \left[\frac{1}{1+\left[\frac{\mathbb{x}}{\mathbb{C}_{\mathbb{m}}^{\mathbb{x}}}\sum_{1\le \mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le \mathbb{m}}\frac{1}{\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{\gamma_{\mathbb{r}_\Xi}}{1-\gamma_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}}, \frac{1}{1+\left[\frac{\mathbb{x}}{\mathbb{C}_{\mathbb{m}}^{\mathbb{x}}}\sum_{1\le \mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le \mathbb{m}}\frac{1}{\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{\varepsilon_{\mathbb{r}_\Xi}}{1-\varepsilon_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}}\right] \\ \left[\frac{1}{1+\left[\frac{\mathbb{x}}{\mathbb{C}_{\mathbb{m}}^{\mathbb{x}}}\sum_{1\le \mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le \mathbb{m}}\frac{1}{\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{\varrho_{\mathbb{r}_\Xi}}{1-\varrho_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}}, \frac{1}{1+\left[\frac{\mathbb{x}}{\mathbb{C}_{\mathbb{m}}^{\mathbb{x}}}\sum_{1\le \mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le \mathbb{m}}\frac{1}{\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{\xi_{\mathbb{r}_\Xi}}{1-\xi_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}}\right] \end{array}\right).$$

Appendix 2: Proof of Theorem 3

$$\overset{\mathbb{x}}{\underset{\Xi=1}{\oplus}} \alpha'_{\mathbb{r}_\Xi} = \left(\begin{array}{c} \left[1-\frac{1}{1+\left(\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{\delta_{\mathbb{r}_\Xi}}{1-\delta_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}\right)^{\frac{1}{\mathcal{L}}}}, 1-\frac{1}{1+\left(\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{\vartheta_{\mathbb{r}_\Xi}}{1-\vartheta_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}\right)^{\frac{1}{\mathcal{L}}}}\right] \\ \left[\frac{1}{1+\left(\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{1-\gamma_{\mathbb{r}_\Xi}}{\gamma_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}\right)^{\frac{1}{\mathcal{L}}}}, \frac{1}{1+\left(\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{1-\varepsilon_{\mathbb{r}_\Xi}}{\varepsilon_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}\right)^{\frac{1}{\mathcal{L}}}}\right] \\ \left[\frac{1}{1+\left(\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{1-\varrho_{\mathbb{r}_\Xi}}{\varrho_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}\right)^{\frac{1}{\mathcal{L}}}}, \frac{1}{1+\left(\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{1-\xi_{\mathbb{r}_\Xi}}{\xi_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}\right)^{\frac{1}{\mathcal{L}}}}\right] \end{array}\right),$$

$$\frac{\overset{\mathbb{x}}{\underset{\Xi=1}{\oplus}} \alpha'_{\mathbb{r}_\Xi}}{\mathbb{x}} = \left(\begin{array}{c} \left[1-\frac{1}{1+\left(\frac{1}{\mathbb{x}}\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{\delta_{\mathbb{r}_\Xi}}{1-\delta_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}\right)^{\frac{1}{\mathcal{L}}}}, 1-\frac{1}{1+\left(\frac{1}{\mathbb{x}}\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{\vartheta_{\mathbb{r}_\Xi}}{1-\vartheta_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}\right)^{\frac{1}{\mathcal{L}}}}\right] \\ \left[\frac{1}{1+\left(\frac{1}{\mathbb{x}}\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{1-\gamma_{\mathbb{r}_\Xi}}{\gamma_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}\right)^{\frac{1}{\mathcal{L}}}}, \frac{1}{1+\left(\frac{1}{\mathbb{x}}\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{1-\varepsilon_{\mathbb{r}_\Xi}}{\varepsilon_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}\right)^{\frac{1}{\mathcal{L}}}}\right] \\ \left[\frac{1}{1+\left(\frac{1}{\mathbb{x}}\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{1-\varrho_{\mathbb{r}_\Xi}}{\varrho_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}\right)^{\frac{1}{\mathcal{L}}}}, \frac{1}{1+\left(\frac{1}{\mathbb{x}}\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{1-\xi_{\mathbb{r}_\Xi}}{\xi_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}\right)^{\frac{1}{\mathcal{L}}}}\right] \end{array}\right),$$

$$\otimes_{1\le \mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le \mathbb{m}}\left(\frac{\overset{\mathbb{x}}{\underset{\Xi=1}{\oplus}}\alpha'_{\mathbb{r}_\Xi}}{\mathbb{x}}\right)=\left(\begin{array}{c}\left[\frac{1}{1+\left[\sum_{1\le \mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le \mathbb{m}}\frac{\mathbb{x}}{\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{\delta_{\mathbb{r}_\Xi}}{1-\delta_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}},1-\frac{1}{1+\left[\sum_{1\le \mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le \mathbb{m}}\frac{\mathbb{x}}{\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{\vartheta_{\mathbb{r}_\Xi}}{1-\vartheta_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}}\right]\\ \left[\frac{1}{1+\left[\sum_{1\le \mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le \mathbb{m}}\frac{\mathbb{x}}{\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{1-\gamma_{\mathbb{r}_\Xi}}{\gamma_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}},\frac{1}{1+\left[\sum_{1\le \mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le \mathbb{m}}\frac{\mathbb{x}}{\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{1-\varepsilon_{\mathbb{r}_\Xi}}{\varepsilon_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}}\right]\\ \left[\frac{1}{1+\left[\sum_{1\le \mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le \mathbb{m}}\frac{\mathbb{x}}{\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{1-\varrho_{\mathbb{r}_\Xi}}{\varrho_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}},\frac{1}{1+\left[\sum_{1\le \mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le \mathbb{m}}\frac{\mathbb{x}}{\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{1-\xi_{\mathbb{r}_\Xi}}{\xi_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}}\right]\end{array}\right).$$

Therefore, we have:

$$IVPFDDHM^{(\mathbb{x})}\left(\varphi'_1,\varphi'_2,\ldots,\varphi'_{\mathbb{m}}\right)=\left(\otimes_{1\le \mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le \mathbb{m}}\left(\frac{\overset{\mathbb{x}}{\underset{\Xi=1}{\oplus}}\varphi'_{\mathbb{r}_\Xi}}{\mathbb{x}}\right)\right)^{\frac{1}{\mathbb{C}_{\mathbb{m}}^{\mathbb{x}}}}=$$

$$\left(\begin{array}{c}\left[\frac{1}{1+\left[\frac{1}{\mathbb{C}_{\mathbb{m}}^{\mathbb{x}}}\sum_{1\le \mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le \mathbb{m}}\frac{\mathbb{x}}{\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{\delta_{\mathbb{r}_\Xi}}{1-\delta_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}},\frac{1}{1+\left[\frac{1}{\mathbb{C}_{\mathbb{m}}^{\mathbb{x}}}\sum_{1\le \mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le \mathbb{m}}\frac{\mathbb{x}}{\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{\vartheta_{\mathbb{r}_\Xi}}{1-\vartheta_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}}\right]\\ \left[1-\frac{1}{1+\left[\frac{1}{\mathbb{C}_{\mathbb{m}}^{\mathbb{x}}}\sum_{1\le \mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le \mathbb{m}}\frac{\mathbb{x}}{\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{1-\gamma_{\mathbb{r}_\Xi}}{\gamma_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}},1-\frac{1}{1+\left[\frac{1}{\mathbb{C}_{\mathbb{m}}^{\mathbb{x}}}\sum_{1\le \mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le \mathbb{m}}\frac{\mathbb{x}}{\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{1-\varepsilon_{\mathbb{r}_\Xi}}{\varepsilon_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}}\right]\\ \left[1-\frac{1}{1+\left[\frac{1}{\mathbb{C}_{\mathbb{m}}^{\mathbb{x}}}\sum_{1\le \mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le \mathbb{m}}\frac{\mathbb{x}}{\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{1-\varrho_{\mathbb{r}_\Xi}}{\varrho_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}},1-\frac{1}{1+\left[\frac{1}{\mathbb{C}_{\mathbb{m}}^{\mathbb{x}}}\sum_{1\le \mathbb{r}_1<\cdots<\mathbb{r}_{\mathbb{x}}\le \mathbb{m}}\frac{\mathbb{x}}{\sum_{\Xi=1}^{\mathbb{x}}\left(\frac{1-\xi_{\mathbb{r}_\Xi}}{\xi_{\mathbb{r}_\Xi}}\right)^{\mathcal{L}}}\right]^{\frac{1}{\mathcal{L}}}}\right]\end{array}\right).$$

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