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Priorities for School Infrastructure Investment in the Face of Uncertainty: An Intuitionistic Fuzzy Multi-Criteria Decision-Making Method Employing the Frank Weighted Averaging Operator

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ABSTRACT

There is a need for a framework of decisions that is able to deal with the uncertainty and subjective nature of expert judgment as well as limited public funds. This paper presents an integrated multi-criteria decision-making approach using Intuitionistic Fuzzy Numbers and the Frank Weighted Averaging operator to prioritize school classroom building projects under a fixed capital budget. Four criteria—safety urgency, number of students affected, cost, and education impact—are measured by an IFN to reflect the membership, non-membership, and hesitancy built into expert assessments, and four candidate schools are assessed against these four criteria. The aggregation of the criteria into one fuzzy score for each alternative is done using a fuzzy operator, IFN-FWA, which is then converted to a crisp matrix using a score function. The obtained matrix is then assessed with five different MCDM methods (TOPSIS, WASPAS, VIKOR, MOORA, and CRITIC) to check the robustness of the ranking. The objective criterion weights are not subjectively assigned but are calculated with the CRITIC method, which will decrease subjectivity in the final recommendation. Although the mathematics involved in each method is different, the results consistently show that the top-ranked school is School4, followed by School2, School1, and School3, with a high level of agreement. The results validate that using the intuitionistic fuzzy aggregation proposed by Frank with multiple validated MCDM approaches can ensure the transparency, defensibility, and stability of the public infrastructure investment decision-making process under uncertainty.

1. Introduction

In 1983 Atanassov extended the concept of fuzzy set theory to intuitionistic fuzzy sets (IFS) in which both membership and non-membership functions are included [1]. An IFS is expressed as an ordered pair of membership and non-membership degrees, and will be called an intuitionistic fuzzy number (IFN) [2].

The ideas of fuzzy numbers were first introduced by Dubois and Prade [3] on the basis of the work of Chang and Zadeh [4]. Burillo *et al.* in 1994 [5] first suggested the generalization to intuitionistic

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fuzzy numbers. Burillo *et al.* proposed two dimensional fuzzy arrays of up-ideal and down-ideal fuzzy numbers, IFNs, to represent IFNs [5].

Intuitionistic fuzzy numbers were developed by several researchers such as K. Atanassov (1983/1986) who introduced IFS, Burillo *et al.* (1994) who first defined IFN, Dubois and Prade (1978) who introduced the concept of fuzzy number, and Xu and Yager (2006) who formalized IFNs with operations [2].

Xu and Yager introduced IFNs with membership and non-membership degrees and hesitancy degree [2]. Triangular IFNs for reliability were introduced by Mahapatra and Roy [6]. Mahapatra and Mahapatra defined trapezoidal IFNs (TrIFN) [6]. Xu presented interval valued IFNs (IVIFN) in [6]. In recent years, intuitionistic fuzzy Z-numbers [7] and intuitionistic trapezoidal fuzzy multi-numbers [8] have been added.

In 1979, Frank proposed the Frank weighted average (FWA) operator as an aggregation operator, which was introduced to replace algebraic, Einstein and Hamacher operators in multi-attribute decision making with uncertain information [9]. The family of continuous triangular norms that satisfy the compatibility condition between the product and the probabilistic sum t-norms is only the family of Frank t-norms, and the parameter they have facilitates more flexibility and robustness in the process of information fusion than other operator families [10].

Frank has proposed an aggregation operator based on t-norms and t-conorms with a parameter $\lambda > 0$ which regulates the influence of membership and non-membership grades in the aggregation process [11]. As the parameter goes to its extreme values, the Frank weighted average operator approaches the basic weighted averaging operator [12] which makes it a generalisation of the basic weighted averaging operator. It is the operator that has desirable mathematical properties such as idempotence, commutativity, boundedness and monotonicity under any fuzzy environments where the operator is formulated [13].

Unlike many other operators based on t-norms, Frank operational laws are generalizations of algebraic, Einstein, and Hamacher operational laws and are therefore more applicable in a larger setting than the latter. The Frank operator has also a lower computational cost than operators like the Bonferroni mean and Heronian mean, and is the only t-norm type that satisfies the compatibility rule [10]. The Frank weighted average was adapted in many fuzzy sets environments such as hesitant fuzzy sets (HFFWA) [14], intuitionistic fuzzy sets (IFFWA / IFFPWA) [13], picture fuzzy sets (PFFWA / IVPFFWA) [15], q-rung orthopair fuzzy sets (Cq-ROFFWA) [10] and interval valued neutrosophic sets (IVNFWA) [16]. In addition, the operator has been extended to complex intuitionistic fuzzy sets (CIFFWA) [9], complex T-spherical fuzzy sets (CT-SFFWA) [11], T-spherical fuzzy sets with softmax integration (T-SFFSWA) [17], q-fractional fuzzy sets (q-FrFFWA) [18], p,q,r-fractional fuzzy sets [19] and interval-valued linear Diophantine fuzzy sets (IVLDFFWA) [20] and quasirung orthopair fuzzy sets (QOFFWA) [21]. Each extension maintains the Frank parameter and adds the membership constraints specific to the environment, with the quasirung orthopair version showing that making the parameters take certain values reproduces different special cases such as the IF, Pythagorean fuzzy and q-rung orthopair weighted averages [21]. TOPSIS is a TOAD method developed by Hwang and Yoon in 1981 to solve multi-criteria decision making problems [22]. It was first introduced in the lecture notes volume *Multiple Attribute Decision Making: Methods and Applications – A State-of-the-Art Survey* (Lecture Notes in Economics and Mathematical Systems, vol. 186, Springer) [23]. It has been consistently cited in multiple sources as a method developed by Hwang and Yoon together, including some research papers that state that the authors are "Yoon and Hwang" and others that state "Hwang and Yoon" [24]. The original formulation from 1981 was further refined by Yoon in 1987, and Hwang *et al.* in 1993 [25]. The model later was extended to fuzzy set theory by Chen, who were the first to apply the model to decision making problems with fuzzy information [26]. Later a Modified TOPSIS was developed where Shannon's entropy theory was introduced to elicit the objective weights for the objectives and a new variant of TOPSIS was proposed [27]. In 1981, C.L. Hwang and K.

Yoon developed this method in the book Multiple Attribute Decision Making [28] (Springer Lecture Notes in Economics and Mathematical Systems, vol. 186). The principal behind the TOPSIS is to choose the alternative that is closest to the positive ideal solution and farthest from the negative ideal solution in the space of multi-criteria/multi-attribute decision making (MCDM/MADM). The basic concept of the method is based on the assumption that the chosen alternative must be closest to the positive ideal solution (which is composed of the best values based on the criteria) and farthest from the negative ideal solution (which is composed of the worst values based on the criteria) [29]. The positive ideal solution is the best points of each criterion and the negative ideal solution combines the worst points. The method exhibits a logical mathematical underpinning, simplicity and easy applicability; and its applications are also well documented with more than 13,000 citations as a top decision making method [27]. Based on the literature review conducted within the period 2000–2015, 105 articles that have expanded and/or modified the technique were identified, among which 49 papers dealt with development or extension of TOPSIS and 56 papers with new modification of the technique [30]. Many extensions of the method have been developed, such as: Three-way decision TOPSIS extension, Pythagorean fuzzy TOPSIS extension, fuzzy TOPSIS and Bayesian TOPSIS extension [31]. MOORA, VIKOR and WASPAS are the important multi-criteria decision-making (MCDM) methods developed by various researchers. The knowledge of their origins gives insight into their applications and relevance in various fields. The MOORA (Multi-Objective Optimization on the Basis of Ratio Analysis) method was developed in 2006 by Brauers and Zavadskas [32]. Uses a ratio system to compare alternatives using several objectives, and it is applicable in a variety of applications including material selection and quality control [32]. VIKOR method proposed in 1979 by S. Opricovic, and named in 1990 [32]. It aims to identify and prioritize options to get the best possible compromise solution, and can be used in social, economic and environmental settings [32]. WASPAS (Weighted Aggregated Sum Product Assessment) is an aggregation-type MCDM method which integrates some characteristics of each additive-type and multiplicative-type methods [33]. The origin of WASPAS is not described in the papers we have, but it is known to be a valuable approach in decision-making processes [33].

2. Preliminaries

This section presents the fundamental definitions related to Intuitionistic Fuzzy Sets (IFS), including the core concept, its special forms, associated numerical measures, and the extension to intuitionistic fuzzy numbers. These definitions form the basis for the theoretical framework used in subsequent analysis.

Definition 2.1 (Intuitionistic Fuzzy Set (IFS)). An **Intuitionistic Fuzzy Set (IFS)** A in a fixed universe X is an object having the form:

$$A = \{ \langle x, \nu_A(x), \Omega_A(x) \rangle \mid x \in X \}$$

where the functions $\nu_A : X \rightarrow [0, 1]$ and $\Omega_A : X \rightarrow [0, 1]$ define the *degree of membership* and the *degree of non-membership* of the element $x \in X$ to the set A , respectively. These functions must satisfy the following condition for every $x \in X$:

$$0 \leq \nu_A(x) + \Omega_A(x) \leq 1.$$

For each element x , the value $\pi_A(x) = 1 - \nu_A(x) - \Omega_A(x)$ is called the **hesitation degree** (or intuitionistic index) of x in A , representing the degree of indeterminacy or lack of knowledge.

Definition 2.2 (Null and Full IFS). The following are two fundamental, extreme cases of an IFS.

- The **Null (or Empty) IFS**, denoted 0_{IFS} , is defined for all $x \in X$ as having no membership and complete non-membership:

$$0_{IFS} = \{\langle x, 0, 1 \rangle \mid x \in X\}.$$

- The **Full IFS**, denoted 1_{IFS} , is defined for all $x \in X$ as having complete membership and no non-membership:

$$1_{IFS} = \{\langle x, 1, 0 \rangle \mid x \in X\}.$$

Definition 2.3 (Score and Accuracy Functions). To compare and rank different intuitionistic fuzzy values (pairs (ν, Ω)), the following functions are commonly used.

- The **Score Function** S for an intuitionistic fuzzy value (ν, Ω) is defined as:

$$S(\nu, \Omega) = \nu - \Omega.$$

The score lies in the interval $[-1, 1]$. A higher score indicates a stronger net membership evidence.

- The **Accuracy Function** H for an intuitionistic fuzzy value (ν, Ω) is defined as:

$$H(\nu, \Omega) = \nu + \Omega.$$

The accuracy lies in the interval $[0, 1]$ and represents the total determinacy (lack of hesitation). A higher accuracy indicates greater certainty in the evaluation.

Given two intuitionistic fuzzy values $a_1 = (\nu_1, \Omega_1)$ and $a_2 = (\nu_2, \Omega_2)$, the ranking rule is:

1. If $S(a_1) > S(a_2)$, then a_1 is greater than a_2 .
2. If $S(a_1) = S(a_2)$, then:
 - (a) If $H(a_1) > H(a_2)$, then a_1 is greater than a_2 .
 - (b) If $H(a_1) = H(a_2)$, then a_1 and a_2 represent the same information.

Definition 2.4 (Intuitionistic Fuzzy Number (IFN)). An **Intuitionistic Fuzzy Number (IFN)** $\tilde{a}$ is a special case of an IFS defined on the real number line $\mathbb{R}$, where the membership and non-membership functions are defined as:

$$\tilde{a} = \{\langle x, \nu_{\tilde{a}}(x), \Omega_{\tilde{a}}(x) \rangle \mid x \in \mathbb{R}\}.$$

Frank operations extend the algebraic behavior of intuitionistic fuzzy numbers through flexible aggregation mechanisms governed by a parameter $\varrho > 1$. They integrate different fuzzy logic operators in a parametric form, allowing interpolation between logical and arithmetic behaviors, thereby increasing flexibility in modeling uncertainty.

Let $\tilde{a}_1 = \langle \mu_1, \nu_1 \rangle$ and $\tilde{a}_2 = \langle \mu_2, \nu_2 \rangle$ be two intuitionistic fuzzy numbers, where $0 \leq \mu_i + \nu_i \leq 1$ for $i = 1, 2$, and let $\varrho > 1$ be the Frank parameter.

Definition 2.5 (Frank Sum (Addition) of IFNs). The Frank sum models the Frank t -conorm aggregation of two IFNs. It generalizes the operation of fuzzy union, where compensation between uncertainties is regulated by ϱ . Larger ϱ values make the operation closer to the probabilistic sum, while smaller values yield more rational OR-like behavior. This operation absorbs additive and logical accumulation of uncertain information.

$$\tilde{a}_1 \oplus_F \tilde{a}_2 = \left\langle 1 - \log_{\varrho} \left(1 + \frac{(\varrho^{1-\nu_1} - 1)(\varrho^{1-\nu_2} - 1)}{\varrho - 1} \right), \log_{\varrho} \left(1 + \frac{(\varrho^{\Omega_1} - 1)(\varrho^{\Omega_2} - 1)}{\varrho - 1} \right) \right\rangle$$

Definition 2.6 (Frank Product (Multiplication) of IFNs). Using the Frank t -norm, the Frank product establishes the intersection or conjunction of two IFNs. It controls the degree of agreement required between two uncertain factors for their intersection to hold. Larger ϱ values incline toward multiplicative behavior, while lower values mirror the minimal operator. This offers a smooth and adaptable approach for simulating fuzzy AND-type relationships.

$$\tilde{a}_1 \otimes_F \tilde{a}_2 = \left\langle \log_{\varrho} \left(1 + \frac{(\varrho^{\nu_1} - 1)(\varrho^{\nu_2} - 1)}{\varrho - 1} \right), 1 - \log_{\varrho} \left(1 + \frac{(\varrho^{1-\Omega_1} - 1)(\varrho^{1-\Omega_2} - 1)}{\varrho - 1} \right) \right\rangle$$

Definition 2.7 (Power Operation for IFNs). The power operation uses Frank algebraic principles to imitate repetitive self-combination of an IFN. It modifies the membership and non-membership degrees via n -governed exponential propagation, enabling sensitivity analysis to ascertain the evolution of uncertainty with iterative aggregation. This process captures the impacts of attenuation or reinforcement in multi-step reasoning.

$$\tilde{a}_1^n = \left\langle \log_{\varrho} \left(1 + \frac{(\varrho^{\nu_1} - 1)^n}{(\varrho - 1)^{n-1}} \right), 1 - \log_{\varrho} \left(1 + \frac{(\varrho^{1-\Omega_1} - 1)^n}{(\varrho - 1)^{n-1}} \right) \right\rangle$$

Definition 2.8 (Scalar Multiplication of IFNs). Scalar multiplication scales the uncertainty levels of an IFN proportionately to n , showing how confidence is strengthened or weakened when evidence is applied repeatedly. This operation is crucial for scaling, normalization, and weighting in multi-criteria decision making models, playing a vital role in building composite fuzzy evaluations.

$$n\tilde{a}_1 = \left\langle 1 - \log_{\varrho} \left(1 + \frac{(\varrho^{1-\nu_1} - 1)^n}{(\varrho - 1)^{n-1}} \right), \log_{\varrho} \left(1 + \frac{(\varrho^{\Omega_1} - 1)^n}{(\varrho - 1)^{n-1}} \right) \right\rangle$$

Definition 2.9 ((FWA Operator)). Let $\alpha_i = (\mu_i, \nu_i)$, for $i = 1, 2, \dots, n$, be a collection of intuitionistic fuzzy numbers (IFNs), and let

$$w = (w_1, w_2, \dots, w_n)^T$$

be their associated weight vector, such that $w_i \in [0, 1]$ and $\sum_{i=1}^n w_i = 1$.

Then the **Frank Weighted Averaging operator** is a mapping $\text{FWA}_w : \Omega^n \rightarrow \Omega$ defined as:

$$\text{FWA}_w(\alpha_1, \alpha_2, \dots, \alpha_n) = w_1\alpha_1 \oplus_F w_2\alpha_2 \oplus_F \dots \oplus_F w_n\alpha_n \quad (1)$$

Definition 2.10 (Aggregated value of IFNFWA operator). Based on the Frank operational laws for IFNs, the aggregated result is given by:

$$\text{FWA}_w(\alpha_1, \dots, \alpha_n) = \left\langle 1 - \log_r \left[1 + \frac{\prod_{i=1}^n (r^{1-\mu_i} - 1)^{w_i}}{(r - 1)^{\sum_{i=1}^n w_i - 1}} \right], \right. \\ \left. \log_r \left[1 + \frac{\prod_{i=1}^n (r^{\nu_i} - 1)^{w_i}}{(r - 1)^{\sum_{i=1}^n w_i - 1}} \right] \right\rangle.$$

3. Problem Statement: Framework for Prioritizing School Infrastructure Projects

The District Education Department faces a resource-allocation problem related to the prioritization of critical school infrastructure investments. Several schools within the district are experiencing classroom overcrowding, while a one-time capital budget of \$150,000 has been allocated to address these infrastructure needs. Since the available funding is insufficient to finance all eligible projects simultaneously, the department must determine which project should receive priority. This decision has direct implications for educational quality, student safety, and the equitable allocation of public resources. At the same time, the allocation process is subject to public scrutiny and must therefore be transparent, analytically justified, and consistent with the district's strategic objectives of equity and educational excellence.

The decision problem is inherently multidimensional because the candidate projects reflect different but equally relevant priorities. A school operating in an older facility may require immediate intervention because of greater safety concerns, whereas a school located in a rapidly growing community may affect a substantially larger number of students and therefore offer a broader impact from the investment. Project costs may also differ considerably depending on the characteristics of the proposed facilities and the required infrastructure. Furthermore, the expected educational impact is not necessarily proportional to either project cost or the number of students affected, since different types of facilities may provide different short- and long-term educational benefits. Consequently, the prioritization problem requires simultaneous consideration of risk mitigation, educational impact, the scale of student benefits, and the efficient use of limited financial resources.

The primary objective is therefore to identify, in a systematic and transparent manner, the highest-priority project among the four eligible school alternatives for classroom infrastructure investment. The decision framework considers four key criteria: Safety Urgency, Number of Students Affected, Cost, and Educational Impact. These criteria represent different dimensions of the investment decision and may involve both quantitative information and qualitative expert judgments. Accordingly, the problem requires an analytical framework capable of integrating heterogeneous criteria while accounting for uncertainty in the evaluation process.

The decision problem can therefore be formulated as a multi-criteria prioritization problem in which four competing school infrastructure projects are evaluated under a fixed budget constraint. The proposed intuitionistic fuzzy MCDM framework is used to aggregate the available information, account for uncertainty and hesitation in expert assessments, and derive a transparent ranking of the alternatives. The resulting prioritization is intended to support the strategic allocation of scarce public resources while providing a clear and defensible analytical basis for the final investment decision.

Table 1
Detailed Criteria Descriptions and Importance

School	Detailed Description and Importance
Safety urgency	Definition: The imminent threat to safety of students and faculty resulting from the current state of the facilities and crowding. Includes building safety, fire safety regulations compliance, evacuation possibilities, and environmental threats. Importance: Moral responsibility to safeguard the physical safety of the learners and the teaching staff Legally mandated compliance with the building code Safety is an indispensable prerequisite to learning Failure to address critical safety concerns can lead to accidents, lawsuits, and closure of the institution

Number of students affected	Definition: Total number of students currently suffering the consequences of overcrowding, along with the number of students who will be helped by having more space. Consists of over-capacity students and those on waiting lists. Importance: Maximizes resources for the greatest number of students Evidences efficient use of public dollars (most bang for the buck) Accounts for equity by taking into account scale of need across the district Reflects shifting demographics and patterns of population growth
Cost	Definition: The total financial investment required for the classroom addition project, including construction, design, permits, and contingency funds. Importance: Limited budget requires careful consideration of costs Fiduciary responsibility to taxpayers for prudent spending Determines feasibility within available funding Opportunity cost: money spent here cannot be used elsewhere
Educational impact	Definition: The new classrooms have the potential to really make a difference in teaching quality. What students learn. This is because the space can be used to help teachers give lessons support special programs and make the whole learning environment better. The classrooms can support programs and this makes the learning environment a great place for students to learn and for teachers to teach. The new classrooms are very important, for student learning outcomes and teaching quality. Importance: Aligns with the goal of schools to educate students Measures how much students improve, not just how much space is available Thinks about the benefits for student success, over a long time Helps with special programs and different ways of teaching

4. MCDM algorithm using (IFN) and IFNFWA operator

4.1 Application of decision making algorithm based on IFFWA

The data supposed data collected in the form of IFN is given in Table 2 for each alternative described in the problem above. By applying the IFNFA operator, we obtained the aggregated values

Table 2

Input data provided by the decision makers

	Safety urgency	Number of students affected	Cost	Educational impact
School ₁	$\langle 0.8, 0.2 \rangle$	$\langle 0.7, 0.3 \rangle$	$\langle 0.6, 0.4 \rangle$	$\langle 0.5, 0.1 \rangle$
School ₂	$\langle 0.5, 0.3 \rangle$	$\langle 0.6, 0.2 \rangle$	$\langle 0.7, 0.1 \rangle$	$\langle 0.9, 0.1 \rangle$
School ₃	$\langle 0.5, 0.1 \rangle$	$\langle 0.5, 0.4 \rangle$	$\langle 0.6, 0.2 \rangle$	$\langle 0.6, 0.4 \rangle$
School ₄	$\langle 0.9, 0.1 \rangle$	$\langle 0.8, 0.2 \rangle$	$\langle 0.9, 0.1 \rangle$	$\langle 0.7, 0.1 \rangle$

given in Table 3.

Table 3

Aggregated Values

Alternatives	Aggregated values using IFNFWA
School ₁	$\langle 0.6669, 0.2235 \rangle$
School ₂	$\langle 0.7173, 0.1575 \rangle$
School ₃	$\langle 0.5524, 0.2408 \rangle$
School ₄	$\langle 0.8425, 0.1192 \rangle$

Now, we apply the score function on the aggregated values obtained in Table 3. The score values obtained for each alternative are given in Table 4. The final ranking of alternatives is given below.

$$\text{School}_4 \succ \text{School}_2 \succ \text{School}_1 \succ \text{School}_3$$

Algorithm 1 IFN-Based Multi-Criteria Decision Making with IFNFWA Operator**1. Step 1: Define the Decision Problem****(a) Identify Alternatives:**

- Let there be m alternatives $A_1, A_2, \dots, A_m$ to be evaluated.

(b) Define Criteria:

- Let there be n criteria $C_1, C_2, \dots, C_n$.

(c) Construct the Decision Matrix:

- Each alternative A_i is evaluated under each School C_j using an IFN:

$$\tilde{a}_{ij} = (\nu_{ij}, \Omega_{ij}), \quad i = 1, \dots, m, \quad j = 1, \dots, n$$

where ν_{ij} is the membership degree, Ω_{ij} is the non-membership degree, and $0 \leq \mu_{ij} + \nu_{ij} \leq 1$.

2. Step 2: Aggregation Using IFNFWA Operator**(a) Assign Weights to Criteria:**

- Let ω_j be the weight for School C_j , where $\sum_{j=1}^n \omega_j = 1$ and $\omega_j \geq 0$.

(b) Apply the IFNFWA Operator:

- For each alternative A_i , compute the aggregated IFN using the IFNFWA operator:

$$IFNFWA(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = \oplus_{j=1}^n (w_j \tilde{a}_j),$$

- This yields a comprehensive IFN for each alternative: $\tilde{A}_i = (\nu_i, \Omega_i)$.

3. Step 3: Compute Score Values**(a) Apply Score Function:**

- For each aggregated IFN $\tilde{A}_i = (\nu_i, \Omega_i)$, compute the score value:

$$S(\tilde{A}_i) = \nu_i - \Omega_i$$

- The score value $S(\tilde{A}_i) \in [-1, 1]$, where higher values indicate better alternatives.

(b) Compute Accuracy Values (if needed for tie-breaking):

- If two alternatives have the same score, compute accuracy values:

$$H(\tilde{A}_i) = \nu_i + \Omega_i$$

- Higher accuracy indicates more reliable information.

4. Step 4: Rank Alternatives and Select the Best Option**(a) Rank Alternatives:**

- Sort alternatives in descending order of their score values $S(\tilde{A}_i)$.
- For equal scores, use accuracy values $H(\tilde{A}_i)$ in descending order.

(b) Final Decision:

- The alternative with the highest ranking is the optimal choice.
- The complete ranking provides a comprehensive decision support:

$$A_{\text{best}} \succ A_{\text{second}} \succ \dots \succ A_{\text{worst}}$$

and rankin show in Figure 1

Table 4
Score function values

Alternative	Score function values
School ₁	0.4434
School ₂	0.5598
School ₃	0.3115
School ₄	0.7233

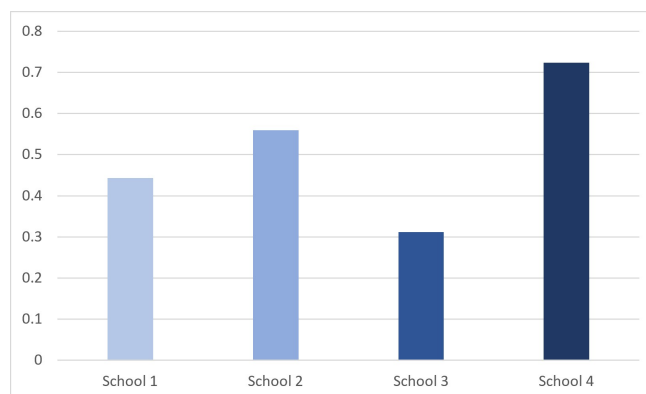


Fig. 1. Grapical Representation of score function

The decision-making process is initiated with the intuitionistic fuzzy assessments provided in Table 2, where each School value is expressed as an intuitionistic fuzzy number $\langle \mu, \nu \rangle$. Following the aggregation of these values using the IFNFWA operator as shown in Table 3, the score function $S = \mu - \nu$ is applied to convert the data into a crisp format. The resulting crisp decision matrix $\mathbf{X}$ is presented in Table 5, where rows represent the four schools and columns correspond to the four evaluation criteria.

Table 5
Crisp Decision Matrix $\mathbf{X}$

Alternative	School ₁	School ₂	School ₃	School ₄
School ₁	0.6667	0.6000	0.0000	0.3333
School ₂	0.0000	0.6000	0.6667	1.0000
School ₃	0.3333	0.0000	0.3333	0.0000
School ₄	1.0000	1.0000	1.0000	0.6667

5. TOPSIS analysis results

The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) is a technique for multi-criteria decision making which utilizes the principle that the selected alternative must be geometrically closest to the positive ideal solution and farthest from the negative ideal solution. The Crisp Decision Matrix $\mathbf{X} = [x_{ij}]_{m \times n}$ is firstly normalized by vector normalization and then weighted using the criteria weights w_j . In this analysis, four alternatives are analyzed for four criteria. Below are the results of computation in steps.

Algorithm 2 Intuitionistic fuzzy TOPSIS procedure for multi-criteria evaluation

- 1: **Step 1: Establish the Decision Framework**
- 2: Specify the alternatives $X_1, X_2, \dots, X_m$ and evaluation criteria $G_1, G_2, \dots, G_n$.
- 3: Form the intuitionistic fuzzy assessment matrix $\tilde{\mathbf{X}} = [\hat{x}_{ij}]_{m \times n}$, where $\hat{x}_{ij} = (p_{ij}, q_{ij})$.
- 4: Ensure that $0 \leq p_{ij} \leq 1$, $0 \leq q_{ij} \leq 1$, and $p_{ij} + q_{ij} \leq 1$.
- 5: **Step 2: Obtain the Normalized IF Matrix**
- 6: For a benefit-type criterion, retain the assessment as $\hat{z}_{ij} = (p_{ij}, q_{ij})$.
- 7: For a cost-type criterion, interchange the membership and non-membership components: $\hat{z}_{ij} = (q_{ij}, p_{ij})$.
- 8: **Step 3: Generate the Weighted IF Matrix**
- 9: Determine criterion weights ω_j satisfying $\omega_j \geq 0$ and $\sum_{j=1}^n \omega_j = 1$.
- 10: Compute the weighted assessment $\hat{y}_{ij} = \omega_j \hat{z}_{ij}$.
- 11: **Step 4: Construct the Reference Ideal Points**
- 12: Define the positive ideal point as $\hat{\Pi}^+ = \{\hat{y}_1^+, \hat{y}_2^+, \dots, \hat{y}_n^+\}$, corresponding to maximum membership and minimum non-membership.
- 13: Define the negative ideal point as $\hat{\Pi}^- = \{\hat{y}_1^-, \hat{y}_2^-, \dots, \hat{y}_n^-\}$, corresponding to minimum membership and maximum non-membership.
- 14: **Step 5: Evaluate Separation Measures**
- 15: Calculate the separation of each alternative from the positive and negative reference points:

$$\Delta_i^+ = \sum_{j=1}^n d(\hat{y}_{ij}, \hat{y}_j^+), \quad \Delta_i^- = \sum_{j=1}^n d(\hat{y}_{ij}, \hat{y}_j^-).$$

- 16: **Step 6: Determine the Relative Closeness Index**

- 17: For every alternative, obtain

$$\Gamma_i = \frac{\Delta_i^-}{\Delta_i^+ + \Delta_i^-}, \quad 0 \leq \Gamma_i \leq 1.$$

- 18: **Step 7: Establish the Final Preference Order**

- 19: Arrange the alternatives according to decreasing values of Γ_i : $\Gamma_{(1)} \geq \Gamma_{(2)} \geq \dots \geq \Gamma_{(m)}$.
- 20: The most preferred alternative is obtained from $X^* = \arg \max_i \Gamma_i$.

5.1 Normalized Decision Matrix

Normalization converts the values of the decision matrix to a common scale through vector normalization method. The normalization value for each decision matrix element x_{ij} is calculated through the use of Euclidean norm formula:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{j=1}^n x_{ij}^2}}, \quad \forall i = 1, 2, \dots, m; \forall j = 1, 2, \dots, n$$

This ensures that all criteria values are dimensionless and range between 0 and 1 for meaningful comparison. The normalization step is crucial as it eliminates the effects of different measurement units across criteria. The resulting normalized decision matrix $\mathcal{R} = [r_{ij}]_{m \times n}$ shown in Table 6 provides the foundation for all subsequent TOPSIS calculations.

Table 6
Normalized Decision Matrix for TOPSIS

Alternative	School ₁	School ₂	School ₃	School ₄
School ₁	0.5345	0.4575	0.0000	0.2672
School ₂	0.0000	0.4575	0.5345	0.8018
School ₃	0.2672	0.0000	0.2672	0.0000
School ₄	0.8018	0.7625	0.8018	0.5345

5.2 Weighted normalized matrix

The weighted normalized matrix is computed by multiplying each normalized value r_{ij} by its corresponding School weight w_j , following the formula:

$$v_{ij} = w_j \times r_{ij}, \quad \text{where } \sum_{j=1}^n w_j = 1$$

This step incorporates the relative importance of each School into the decision-making process. The weights $\omega = [w_1, w_2, \dots, w_n]$ reflect the decision-maker's preferences and priorities for different criteria. Each column in this matrix represents the contribution of a specific School to the overall performance of each alternative. Table 7 presents the complete weighted normalized matrix $\mathcal{V} = [v_{ij}]_{m \times n}$ with values that will be used to determine the ideal solutions.

Table 7
Weighted Normalized Matrix $\mathcal{V}$

Alternative	School ₁	School ₂	School ₃	School ₄
School ₁	0.1336	0.1144	0.0000	0.0668
School ₂	0.0000	0.1144	0.1336	0.2004
School ₃	0.0668	0.0000	0.0668	0.0000
School ₄	0.2004	0.1906	0.2004	0.1336

5.3 Ideal Best and Worst Solutions

The ideal best solution $\mathcal{A}^+$ represents the most favorable values for each School, while the ideal worst solution $\mathcal{A}^-$ represents the least favorable values. For benefit criteria, the best value is the maximum and the worst is the minimum across all alternatives in the weighted normalized matrix:

$$\mathcal{A}^+ = \{v_1^+, v_2^+, \dots, v_n^+\} = \left\{ \left(\max_i v_{ij} \mid j \in \mathcal{J} \right), \left(\min_i v_{ij} \mid j \in \mathcal{J}' \right) \right\}$$

$$\mathcal{A}^- = \{\mathbf{v}_1^-, \mathbf{v}_2^-, \dots, \mathbf{v}_n^-\} = \left\{ \left(\min_i \mathbf{v}_{ij} \mid j \in \mathcal{J} \right), \left(\max_i \mathbf{v}_{ij} \mid j \in \mathcal{J}' \right) \right\}$$

where $\mathcal{J}$ is the set of benefit criteria and $\mathcal{J}'$ is the set of cost criteria. These ideal solutions serve as reference points for calculating the separation measures. The ideal solutions provide a benchmark against which all alternatives are compared. Table 8 displays both the ideal best $\mathcal{A}^+$ and ideal worst $\mathcal{A}^-$ solutions for all four criteria.

Table 8
Ideal Best and Worst Solutions for TOPSIS

	School ₁	School ₂	School ₃	School ₄
Ideal Best ($\mathcal{A}^+$)	0.2004	0.1906	0.2004	0.2004
Ideal Worst ($\mathcal{A}^-$)	0.0000	0.0000	0.0000	0.0000

5.4 Separation measures

The separation of each alternative from the ideal best ($\mathbf{S}_i^+$) and ideal worst ($\mathbf{S}_i^-$) is calculated using the n-dimensional Euclidean distance. The separation from the ideal best is computed as:

$$\mathbf{S}_i^+ = \sqrt{\sum_{j=1}^n (\mathbf{v}_{ij} - \mathbf{v}_j^+)^2}, \quad \forall i = 1, 2, \dots, m$$

while the separation from the ideal worst is:

$$\mathbf{S}_i^- = \sqrt{\sum_{j=1}^n (\mathbf{v}_{ij} - \mathbf{v}_j^-)^2}, \quad \forall i = 1, 2, \dots, m$$

These distance measures quantify how far each alternative is from the positive and negative ideal solutions. Smaller $\mathbf{S}_i^+$ and larger $\mathbf{S}_i^-$ values indicate better alternatives. These separation measures form the basis for calculating the relative closeness coefficients in the final ranking step.

5.5 Closeness coefficient and final ranking

The relative closeness coefficient $\mathbf{C}_i$ measures the similarity of each alternative to the ideal best solution and is computed using:

$$\mathbf{C}_i = \frac{\mathbf{S}_i^-}{\mathbf{S}_i^+ + \mathbf{S}_i^-}, \quad \text{where } 0 \leq \mathbf{C}_i \leq 1$$

This coefficient represents the overall performance score of each alternative, with higher values indicating better alternatives. The final ranking is determined by sorting the alternatives in descending order of their closeness coefficients $\mathbf{C}_i$. Table 9 presents the calculated closeness coefficients $\mathbf{C} = [\mathbf{C}_1, \mathbf{C}_2, \dots, \mathbf{C}_m]$ and the corresponding ranks for all four alternatives and the graphical representation show in Figure 2.

6. Waspas method calculation

This report presents a step-by-step application of the WASPAS (Weighted Aggregated Sum Product Assessment) method for ranking alternatives. The decision matrix, normalized matrix, criteria weights, and intermediate scores for WSM, WPM, and WASPAS are computed. The final ranking identifies the best alternative.

6.1 Decision Matrix

The initial decision matrix (Table 5) contains performance values of four alternatives across four criteria.

Algorithm 3 Intuitionistic fuzzy WASPAS-based decision-making framework

1: **Stage 1: Establishment of the Decision Structure**

2: Define the alternative set $\mathcal{X} = \{X_1, X_2, \dots, X_m\}$ and criterion set $\mathcal{G} = \{G_1, G_2, \dots, G_n\}$.

3: Construct the intuitionistic fuzzy information matrix $\xi = [\xi_{ij}]_{m \times n}$, where $\xi_{ij} = (\alpha_{ij}, \beta_{ij})$ satisfies $0 \leq \alpha_{ij}, \beta_{ij} \leq 1$ and $\alpha_{ij} + \beta_{ij} \leq 1$.

4: **Stage 2: Criterion-Oriented Transformation**

5: Transform each intuitionistic fuzzy assessment according to the nature of criterion G_j :

$$\eta_{ij} = \begin{cases} (\alpha_{ij}, \beta_{ij}), & G_j \text{ is a benefit criterion,} \\ (\beta_{ij}, \alpha_{ij}), & G_j \text{ is a cost criterion.} \end{cases}$$

6: Let ϖ_j denote the normalized importance of criterion G_j , satisfying $\varpi_j \geq 0$ and $\sum_{j=1}^n \varpi_j = 1$.

7: Generate the weighted intuitionistic fuzzy information:

$$\zeta_{ij} = \varpi_j \eta_{ij}.$$

8: **Stage 3: Dual WASPAS Aggregation**

9: Determine the additive preference component:

$$\mathcal{U}_i = \bigoplus_{j=1}^n \zeta_{ij}.$$

10: Determine the multiplicative preference component:

$$\mathcal{V}_i = \bigotimes_{j=1}^n (\eta_{ij})^{\varpi_j}.$$

11: **Stage 4: Score Extraction**

12: Compute the score corresponding to the additive component:

$$\mathcal{E}_i^+ = \mu_{\mathcal{U}_i} - \nu_{\mathcal{U}_i}.$$

13: Compute the score corresponding to the multiplicative component:

$$\mathcal{E}_i^\times = \mu_{\mathcal{V}_i} - \nu_{\mathcal{V}_i}.$$

14: **Stage 5: WASPAS Synthesis**

15: Fuse the additive and multiplicative preference measures using the parameter λ :

$$\mathcal{R}_i(\lambda) = \lambda \mathcal{E}_i^+ + (1 - \lambda) \mathcal{E}_i^\times, \quad \lambda \in [0, 1].$$

16: **Stage 6: Alternative Prioritization**

17: Evaluate $\mathcal{R}_i(\lambda)$ and arrange all alternatives in descending order of their resulting preference values.

18: For sensitivity analysis, repeat the procedure over the selected range of λ and examine changes in the preference order.

19: Determine the optimal alternative according to

$$X^* = \arg \max_{X_i \in \mathcal{X}} \mathcal{R}_i(\lambda).$$

Table 9
Closeness Coefficient and Final Ranking for TOPSIS

Alternative	Closeness Coefficient (C_i)	Rank
School ₁	0.4186	3
School ₂	0.5428	2
School ₃	0.2200	4
School ₄	0.8459	1

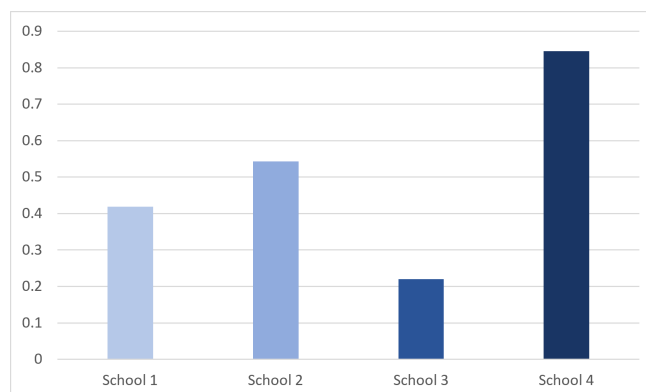


Fig. 2. Graphical representation of TOPSIS ranking

6.2 Normalized decision matrix

All criteria are beneficial, so normalization is performed by dividing each value by the maximum in its column. Since the maximum for each column is 1, the normalized matrix remains identical to the original (Table 5).

6.3 Criteria weights

Equal weights are assigned to all four criteria. Each weight is 0.25.

6.4 WSM, WPM, and WASPAS scores

The WSM score for each alternative is computed as:

$$Q_{1i} = \sum_{j=1}^n w_j \cdot \bar{x}_{ij}$$

The WPM score is computed as:

$$Q_{2i} = \prod_{j=1}^n (\bar{x}_{ij})^{w_j}$$

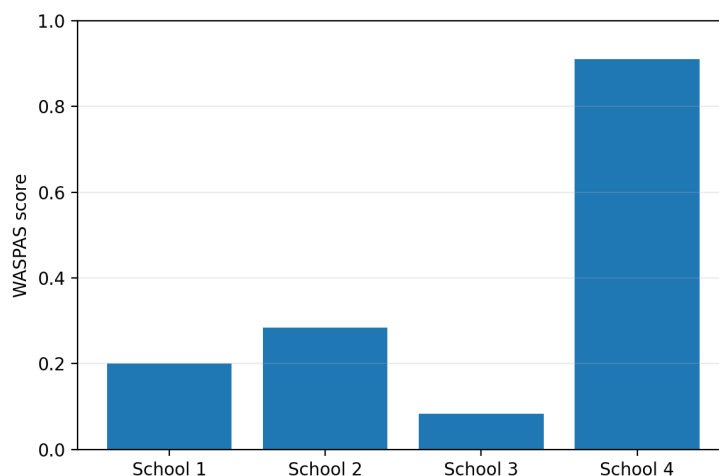
The WASPAS score combines both methods with a parameter $\lambda = 0.5$:

$$Q_i = 0.5 \cdot Q_{1i} + 0.5 \cdot Q_{2i}$$

All three scores are presented together in Table 10 for easy comparison and the result shown in Figure 3.

Table 10
WSM, WPM, and WASPAS Scores

Alternative	WSM Score (Q_{1i})	WPM Score (Q_{2i})	WASPAS Score (Q_i)
School ₁	0.4000	0.0000	0.2000
School ₂	0.5667	0.0000	0.2833
School ₃	0.1666	0.0000	0.0833
School ₄	0.9167	0.9036	0.9101

**Fig. 3.** Ranking of WSM, WPM, and WASPAS Scores

6.5 Ranking of alternatives

Alternatives are ranked in descending order of WASPAS score (1 = best, 4 = worst). Based on the scores from Table 10, the final ranking is presented in Table 11.

Table 11
Final Ranking for
WASPAS

Alternative	Rank
School ₁	3
School ₂	2
School ₃	4
School ₄	1

From Table 10 and Table 11, it is evident that School₄ achieves the highest WASPAS score (0.9101) and is therefore ranked first (best). School₂ follows in second place, School₁ in third, and School₃ is ranked last (worst).

7. Vikor method calculation

This report presents a comprehensive application of the VIKOR (VIšekriterijumsko KOmpromisno Rangiranje) method for ranking four schools (School₁, School₂, School₃, and School₄) based on multiple criteria. The VIKOR method introduces compromise ranking based on the closeness to the ideal solution. The decision matrix (Table 5) and ideal solutions (following the same approach as in Table 8) are used. Utility and regret measures, VIKOR index, and final rankings are computed. Additionally, reciprocal normalization results are presented for sensitivity analysis.

Algorithm 4 Intuitionistic fuzzy VIKOR approach for multi-criteria decision analysis

- 1: **Step 1: Establish the Decision Framework**
- 2: Specify m alternatives $A_1, A_2, \dots, A_m$ and n evaluation criteria $C_1, C_2, \dots, C_n$.
- 3: Form the intuitionistic fuzzy decision matrix:
- 4: $\tilde{x}_{ij} = (\mu_{ij}, \nu_{ij})$
- 5: where $0 \leq \mu_{ij}, \nu_{ij} \leq 1$ and $0 \leq \mu_{ij} + \nu_{ij} \leq 1$.
- 6: **Step 2: Identify the Positive and Negative Ideal Solutions**
- 7: For benefit-type criteria:
- 8: $\tilde{x}_j^{\oplus} = \max_i \tilde{x}_{ij}$
- 9: $\tilde{x}_j^{\ominus} = \min_i \tilde{x}_{ij}$
- 10: where the positive ideal corresponds to maximum membership and minimum non-membership.
- 11: For cost-type criteria:
- 12: $\tilde{x}_j^{\oplus} = \min_i \tilde{x}_{ij}$
- 13: $\tilde{x}_j^{\ominus} = \max_i \tilde{x}_{ij}$.
- 14: **Step 3: Evaluate the Distances from Ideal Solutions**
- 15: Distance of alternative A_i from the positive ideal:
- 16:
$$d_{ij}^{\oplus} = \sqrt{\frac{1}{2} \left[(\mu_{ij} - \mu_j^{\oplus})^2 + (\nu_{ij} - \nu_j^{\oplus})^2 + (\pi_{ij} - \pi_j^{\oplus})^2 \right]}$$
- 17: Distance of alternative A_i from the negative ideal:
- 18:
$$d_{ij}^{\ominus} = \sqrt{\frac{1}{2} \left[(\mu_{ij} - \mu_j^{\ominus})^2 + (\nu_{ij} - \nu_j^{\ominus})^2 + (\pi_{ij} - \pi_j^{\ominus})^2 \right]}$$
- 19: **Step 4: Determine the Utility and Regret Measures**
- 20: Calculate the overall utility measure:
- 21:
$$P_i = \sum_{j=1}^n w_j \frac{d_{ij}^{\oplus}}{d(\tilde{x}_j^{\ominus}, \tilde{x}_j^{\oplus})}$$
- 22: Calculate the maximum individual regret:
- 23:
$$T_i = \max_j \left[w_j \frac{d_{ij}^{\oplus}}{d(\tilde{x}_j^{\ominus}, \tilde{x}_j^{\oplus})} \right]$$
- 24: where w_j denotes the weight assigned to criterion C_j and $\sum_{j=1}^n w_j = 1$.
- 25: **Step 5: Calculate the Compromise Measure**
- 26: Determine the VIKOR compromise index:
- 27:
$$V_i = v \frac{P_i - P^{\min}}{P^{\max} - P^{\min}} + (1 - v) \frac{T_i - T^{\min}}{T^{\max} - T^{\min}}$$
- 28: where $P^{\max} = \max_i P_i$, $P^{\min} = \min_i P_i$, $T^{\max} = \max_i T_i$, and $T^{\min} = \min_i T_i$.
- 29: The parameter $v \in [0, 1]$ represents the weight assigned to the majority criterion.
- 30: **Step 6: Establish the Ranking**
- 31: Arrange the alternatives in ascending order according to V_i , P_i , and T_i .
- 32: The alternative associated with the smallest V_i is regarded as the preferred solution.
- 33: **Step 7: Determine the Compromise Solution**
- 34: Let $A^{(1)}$ and $A^{(2)}$ denote the first and second alternatives in the ascending order of V_i .
- 35: Verify the following two conditions:
- 36: C1: $V(A^{(2)}) - V(A^{(1)}) \geq \frac{1}{m-1}$
- 37: C2: $A^{(1)}$ occupies the first position in either the P or T ranking.
- 38: If both C1 and C2 are satisfied, $A^{(1)}$ is selected as the compromise solution.
- 39: Otherwise, the alternatives satisfying the VIKOR compromise conditions are retained as the compromise set.

7.1 Decision matrix

The initial decision matrix (Table 5) contains performance values of four schools across four criteria.

7.2 Ideal best and worst Values

For beneficial criteria, the ideal best (f_j^*) and ideal worst (f_j^-) values are determined as the maximum and minimum values across all schools for each criterion, respectively. These follow the same pattern as the TOPSIS ideal solutions shown in Table 8. The ideal best values are all 1, and ideal worst values are all 0 for each criterion.

7.3 Utility measure (S)

The utility measure (S_i) represents the group benefit and is computed as:

$$S_i = \sum_{j=1}^n w_j \frac{(f_j^* - x_{ij})}{(f_j^* - f_j^-)}$$

For this analysis, weights are assumed equal ($w_j = 0.25$). The results are shown in Table 12.

Table 12
Utility Measure (S)
Values for VIKOR

School	S_i
School ₁	0.6000
School ₂	0.4333
School ₃	0.8334
School ₄	0.0833

7.4 Regret measure (R)

The regret measure (R_i) represents the individual regret and is computed as:

$$R_i = \max_j \left[w_j \frac{(f_j^* - x_{ij})}{(f_j^* - f_j^-)} \right]$$

The results are presented in Table 13.

Table 13
Regret Measure (R)
Values for VIKOR

School	R_i
School ₁	0.2500
School ₂	0.2500
School ₃	0.2500
School ₄	0.0833

7.5 VIKOR Index (Q)

The VIKOR index (Q_i) combines the utility and regret measures using a compromise factor $v = 0.5$:

$$Q_i = v \frac{(S_i - S^*)}{(S^- - S^*)} + (1 - v) \frac{(R_i - R^*)}{(R^- - R^*)}$$

Table 16
Reciprocal
normalization scores
for VIKOR ($\lambda = 0.9$)

School	Score
School ₁	0.2031
School ₂	0.2169
School ₃	0.1864
School ₄	0.3936

where $S^* = \min S_i$, $S^- = \max S_i$, $R^* = \min R_i$, and $R^- = \max R_i$. The results are shown in Table 14.

Table 14
VIKOR Index (Q)
Values

School	Q_i
School ₁	0.8444
School ₂	0.7333
School ₃	1.0000
School ₄	0.0000

7.6 Rankings by S, R, and Q

Based on the scores from Tables 12, 13, and 14, rankings are assigned (1 = best, 4 = worst). These rankings are presented in Table 15.

Table 15
Rankings by Utility (S), Regret (R), and VIKOR Index (Q)

School	Ranking by S	Ranking by R	Ranking by Q
School ₁	3	2	3
School ₂	2	2	2
School ₃	4	2	4
School ₄	1	1	1

7.7 VIKOR Reciprocal Normalization Results

For sensitivity analysis, reciprocal normalization was applied with a parameter $\lambda = 0.9$. The resulting scores are presented in Table 16 and shown in Figure 4.

Based on the VIKOR index values from Table 14, School₄ has the lowest Q value (0.0000) and is therefore ranked first (best), followed by School₂ (0.7333), School₁ (0.8444), and School₃ (1.0000) as shown in Table 15. The reciprocal normalization results from Table 16 provide additional insight into the stability of the ranking under different normalization schemes, with School₄ again showing the highest score (0.3936).

8. Moora method calculation

This report presents a comprehensive application of the MOORA (Multi-Objective Optimization by Ratio Analysis) method for ranking four schools (School₁, School₂, School₃, and School₄) based on multiple criteria. The MOORA method involves ratio analysis to determine the best alternative by maximizing beneficial criteria. The step-by-step procedure includes constructing the decision matrix, normalization, weighting, computing overall performance scores, and final ranking.

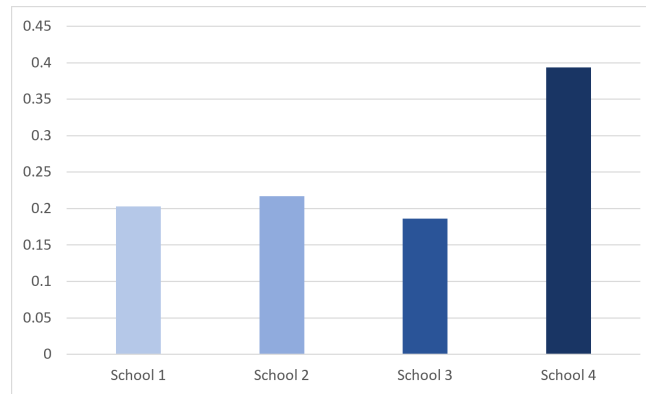


Fig. 4. Graphical representation of Vikor ranking

Algorithm 5 Intuitionistic fuzzy MOORA method for multi-criteria decision making

- 1: **Step 1: Define the Decision Problem**
 - 2: Identify m alternatives $A_1, A_2, \dots, A_m$ and n criteria $C_1, C_2, \dots, C_n$
 - 3: Construct intuitionistic fuzzy decision matrix: $\tilde{x}_{ij} = (\mu_{ij}, \nu_{ij})$
 - 4: where $0 \leq \mu_{ij}, \nu_{ij} \leq 1$ and $0 \leq \mu_{ij} + \nu_{ij} \leq 1$
 - 5: **Step 2: Normalize the Decision Matrix**
 - 6: Calculate the normalized values using vector normalization:
 - 7:
$$\tilde{r}_{ij} = \frac{\tilde{x}_{ij}}{\sqrt{\sum_{i=1}^m \tilde{x}_{ij}^2}} = \left(\frac{\mu_{ij}}{\sqrt{\sum_{i=1}^m \mu_{ij}^2}}, \frac{\nu_{ij}}{\sqrt{\sum_{i=1}^m \nu_{ij}^2}} \right)$$
 - 8: Alternative normalization:
 - 9:
$$\tilde{r}_{ij} = \frac{\tilde{x}_{ij}}{\max_i \tilde{x}_{ij}} = \left(\frac{\mu_{ij}}{\max_i \mu_{ij}}, \frac{\nu_{ij}}{\max_i \nu_{ij}} \right)$$
 - 10: **Step 3: Calculate the Weighted Normalized Matrix**
 - 11: Assign weights $w_j \geq 0$ with $\sum_{j=1}^n w_j = 1$
 - 12: Calculate: $\tilde{v}_{ij} = w_j \cdot \tilde{r}_{ij} = (1 - (1 - \mu_{r_{ij}})^{w_j}, \nu_{r_{ij}}^{w_j})$
 - 13: **Step 4: Determine Criteria Types**
 - 14: Identify benefit criteria (to be maximized) and cost criteria (to be minimized)
 - 15: **Step 5: Calculate the Overall Performance Index**
 - 16: For each alternative, sum the weighted normalized values:
 - 17:
$$\tilde{y}_i = \sum_{j=1}^g \tilde{v}_{ij} - \sum_{j=g+1}^n \tilde{v}_{ij}$$
 - 18: where g is the number of benefit criteria
 - 19: $\tilde{y}_i = (\mu_{y_i}, \nu_{y_i})$ using IF addition and subtraction operations
 - 20: **Step 6: Calculate Score Values**
 - 21: $S(\tilde{y}_i) = \mu_{y_i} - \nu_{y_i}$
 - 22: Alternatives with higher $S(\tilde{y}_i)$ values are preferred
 - 23: **Step 7: Rank Alternatives**
 - 24: Rank by descending $S(\tilde{y}_i)$: $S(\tilde{y}_{(1)}) \geq S(\tilde{y}_{(2)}) \geq \dots \geq S(\tilde{y}_{(m)})$
 - 25: Select: $A_{\text{best}} = \arg \max_i S(\tilde{y}_i)$
-

8.1 Initial decision matrix

The initial decision matrix (Table 5) contains performance values of four schools across four criteria. All criteria are considered beneficial (to be maximized).

8.2 Normalized decision matrix

Vector normalization is performed using the formula:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^m x_{ij}^2}}$$

The normalized decision matrix is presented in Table 17.

Table 17

Normalized Decision Matrix for MOORA (Vector Normalization)

School	Criteria			
	C1	C2	C3	C4
School ₁	0.5345	0.4575	0.0000	0.2672
School ₂	0.0000	0.4575	0.5345	0.8018
School ₃	0.2672	0.0000	0.2672	0.0000
School ₄	0.8018	0.7625	0.8018	0.5345

8.3 Weighted normalized matrix

The weighted normalized matrix is obtained by multiplying each normalized value by its corresponding weight:

$$v_{ij} = r_{ij} \times w_j$$

Equal weights of 0.25 are assigned to all criteria, as shown in Table 18.

Table 18

Weighted Normalized Matrix for MOORA
($w_j = 0.25$)

School	Criteria			
	C1	C2	C3	C4
School ₁	0.1336	0.1144	0.0000	0.0668
School ₂	0.0000	0.1144	0.1336	0.2004
School ₃	0.0668	0.0000	0.0668	0.0000
School ₄	0.2004	0.1906	0.2004	0.1336

8.4 Overall performance scores

The overall performance score for each alternative is computed as:

$$y_i = \sum_{j=1}^n v_{ij} \quad (\text{for beneficial criteria})$$

Since all criteria are beneficial, the scores are simply the sum of weighted normalized values. Results are shown in Table 19.

Table 19
Overall Performance
Scores for MOORA

School	Score (y_i)
School ₁	0.3148
School ₂	0.4485
School ₃	0.1336
School ₄	0.7251

8.5 Final ranking

Based on the scores from Table 19, schools are ranked in descending order (higher score = better). Table 20 presents the final ranking and graph shown in Figure 5.

Table 20
Final Ranking of Schools
using MOORA

Rank	School	Score
1	School ₄	0.7251
2	School ₂	0.4485
3	School ₁	0.3148
4	School ₃	0.1336

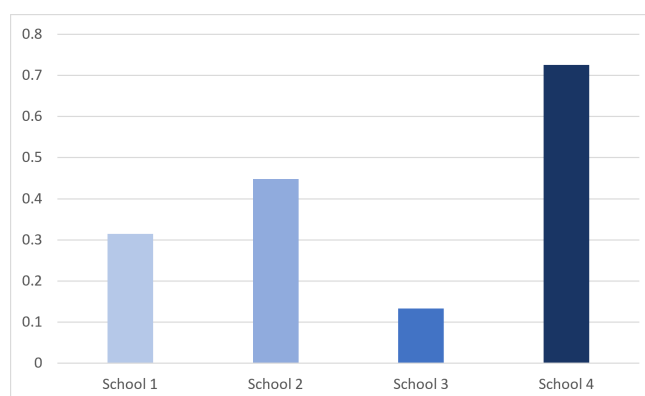


Fig. 5. Ranking of MOORA method

CRITIC method evaluation steps

Step 3: Normalized Decision Matrix

To account for different units and scales, the initial decision matrix is normalized into a dimensionless matrix $X^* = [x_{ij}^*]_{4 \times 4}$, where each value $x_{ij}^* \in [0, 1]$ represents the relative performance of Alternative i under School j . The resulting normalized matrix is shown in Table 21.

Step 4: Standard deviation for each school

The standard deviation (σ_j) measures the contrast intensity (dispersion) of each School across all alternatives. Higher variation indicates greater informational weight; the calculated values are listed in Table 22.

Step 5: Correlation Matrix

The linear correlation matrix $R = [r_{jk}]_{4 \times 4}$ quantifies the degree of conflict and interdependency between each pair of criteria j and k , as detailed in Table 23.

Algorithm 6 Intuitionistic fuzzy CRITIC-based criteria weighting procedure

1: **Step 1: Initialize the Decision Environment**

2: Define the alternative set $\mathcal{A} = \{A_1, A_2, \dots, A_m\}$ and criterion set $\mathcal{C} = \{C_1, C_2, \dots, C_n\}$.

3: Form the intuitionistic fuzzy evaluation matrix $\mathbf{F} = [\xi_{ij}]_{m \times n}$, where $\xi_{ij} = (\alpha_{ij}, \beta_{ij})$.

4: The intuitionistic fuzzy degrees satisfy $0 \leq \alpha_{ij} \leq 1$, $0 \leq \beta_{ij} \leq 1$, and $\alpha_{ij} + \beta_{ij} \leq 1$.

5: **Step 2: Transform the Evaluation Information**

6: Normalize the intuitionistic fuzzy assessments according to the type of criterion:

$$\eta_{ij} = \begin{cases} \left(\frac{\alpha_{ij} - \alpha_j^{\min}}{\alpha_j^{\max} - \alpha_j^{\min}}, \frac{\beta_{ij} - \beta_j^{\min}}{\beta_j^{\max} - \beta_j^{\min}} \right), & C_j \text{ is a benefit criterion,} \\ \left(\frac{\alpha_j^{\max} - \alpha_{ij}}{\alpha_j^{\max} - \alpha_j^{\min}}, \frac{\beta_j^{\max} - \beta_{ij}}{\beta_j^{\max} - \beta_j^{\min}} \right), & C_j \text{ is a cost criterion.} \end{cases}$$

7: **Step 3: Derive the score information**

8: Calculate the score corresponding to each normalized intuitionistic fuzzy assessment:

$$\chi_{ij} = \mu_{\eta_{ij}} - \nu_{\eta_{ij}}.$$

9: Hence, $\chi_{ij} \in [-1, 1]$.

10: **Step 4: Establish the Inter-Criterion Correlation**

11: Determine the Pearson correlation coefficient between criteria C_j and C_k as

$$\varrho_{jk} = \frac{\sum_{i=1}^m (\chi_{ij} - \bar{\chi}_j)(\chi_{ik} - \bar{\chi}_k)}{\sqrt{\sum_{i=1}^m (\chi_{ij} - \bar{\chi}_j)^2 \sum_{i=1}^m (\chi_{ik} - \bar{\chi}_k)^2}}.$$

12: The mean score of criterion C_j is

$$\bar{\chi}_j = \frac{1}{m} \sum_{i=1}^m \chi_{ij}.$$

13: **Step 5: Measure the Contrast Intensity**

14: Compute the standard deviation associated with criterion C_j :

$$\delta_j = \sqrt{\frac{1}{m} \sum_{i=1}^m (\chi_{ij} - \bar{\chi}_j)^2}.$$

15: **Step 6: Quantify the Information Content**

16: Determine the information contribution of criterion C_j using

$$\mathcal{I}_j = \delta_j \sum_{k=1}^n (1 - \varrho_{jk}).$$

17: Here, $\mathcal{I}_j$ denotes the total information provided by criterion C_j .

Algorithm 6 Intuitionistic fuzzy CRITIC-based criteria weighting procedure (continued)1: **Step 7: Obtain the Objective Criterion Weights**

2: Normalize the information quantities to obtain

$$\omega_j = \frac{\mathcal{I}_j}{\sum_{k=1}^n \mathcal{I}_k}.$$

3: The resulting weights satisfy $\omega_j \geq 0$ and $\sum_{j=1}^n \omega_j = 1$.4: **Step 8: Generate the Weight Vector**

5: Return the objective criterion-weight vector

$$\omega = [\omega_1, \omega_2, \dots, \omega_n]^T.$$

Table 21

Normalized Decision Matrix for CRITIC

Alternative	School ₁	School ₂	School ₃	School ₄
School ₁	0.6667	0.6000	0.0000	0.3333
School ₂	0.0000	0.6000	0.6667	1.0000
School ₃	0.3333	0.0000	0.3333	0.0000
School ₄	1.0000	1.0000	1.0000	0.6667

Table 22Standard Deviation (σ_j) for Each School (CRITIC)

School	Standard Deviation (σ_j)
School ₁	0.4303
School ₂	0.4123
School ₃	0.4303
School ₄	0.4303

Table 23Intercriteria Correlation Matrix (R) for CRITIC

	School ₁	School ₂	School ₃	School ₄
School ₁	1.0000	0.5636	0.2000	-0.2000
School ₂	0.5636	1.0000	0.5636	0.6888
School ₃	0.2000	0.5636	1.0000	0.6000
School ₄	-0.2000	0.6888	0.6000	1.0000

Step 6: Quantity of information for each school

The measure of information contained in each School (C_j) combines both contrast intensity (σ_j) and conflict with other criteria $\sum_{k=1}^m (1 - r_{jk})$ according to $C_j = \sigma_j \sum_{k=1}^m (1 - r_{jk})$. The calculated measures are summarized in Table 24.

The graphical representation of the information quantities (C_j) for the four schools is presented in Figure 6.

Table 24
Quantity of Information (C_j) for Each School
(CRITIC)

School	Information Quantity (C_j)
School ₁	0.8763
School ₂	0.4882
School ₃	0.7042
School ₄	0.6503

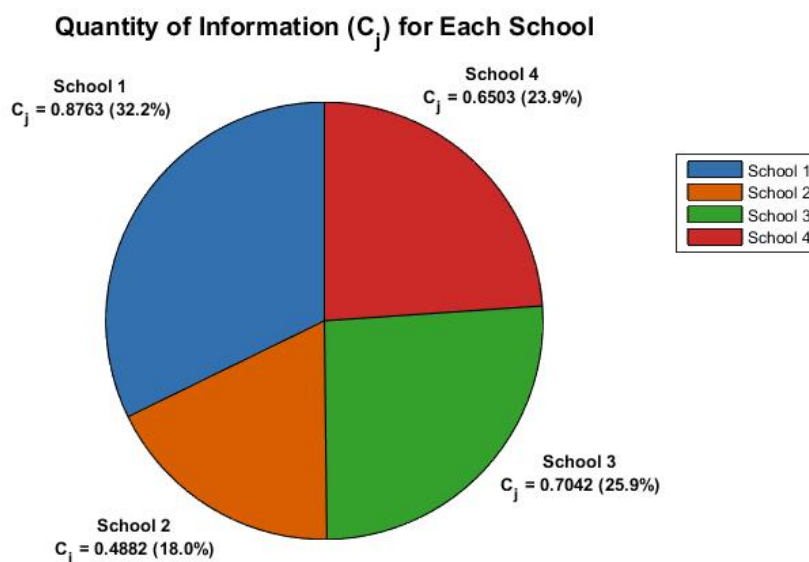


Fig. 6. Information Quantity of Schools Based on the CRITIC Method

Step 7: Objective criteria weights

The relative objective weight (w_j) for each School is calculated by normalizing the information quantity C_j such that $\sum_{j=1}^m w_j = 1$. The resulting weights are presented in Table 25.

Table 25
Objective Criteria Weights (w_j) from CRITIC

School (j)	Description	Weight (w_j)
School ₁	Safety urgency	0.3223
School ₂	Number of students affected	0.1795
School ₃	Cost	0.2590
School ₄	Educational impact	0.2392

Step 8 & 9: Final performance scores and rankings

By aggregating the normalized performance values x_{ij}^* with their corresponding objective weights w_j , the overall preference score $S_i = \sum_{j=1}^m w_j x_{ij}^*$ for each school is obtained. As shown in Table 26, School₄ achieves the highest score, while School₃ ranks last and graph shown in Figure 7.

Table 26
Final Performance Scores (S_i) and
Rankings of Schools (CRITIC)

Rank	School	Final Score (S_i)
1	School ₄	0.9203
2	School ₂	0.5196
3	School ₁	0.4023
4	School ₃	0.1938

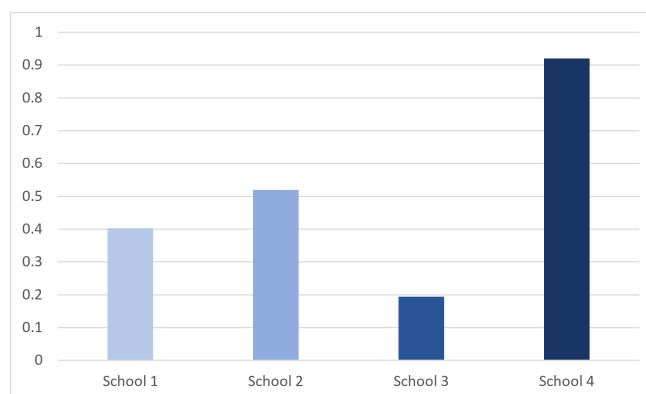


Fig. 7. Ranking of CRITIC method

9. Comparative analysis

9.1 Summary of rankings across all methods

Based on the comprehensive analysis presented in the document, all six MCDM methods consistently identified School₄ as the optimal choice, followed by School₂, School₁, and School₃. This consistency demonstrates the robustness of the proposed decision-making framework. The comparative results are systematically presented in Table 27, which displays the performance scores obtained from each method alongside the corresponding rankings and Figure 8 show the graphical representation of Table 27.

Table 27
Comparison of MCDM Methods and Final Performance Scores

Method	School ₁	School ₂	School ₃	School ₄	Ranking
INFFWA Score	0.4434	0.5598	0.3115	0.7233	$S_4 \succ S_2 \succ S_1 \succ S_3$
TOPSIS	0.4186	0.5428	0.2200	0.8459	$S_4 \succ S_2 \succ S_1 \succ S_3$
WASPAS	0.2000	0.2833	0.0833	0.9101	$S_4 \succ S_2 \succ S_1 \succ S_3$
VIKOR	0.2031	0.2169	0.1864	0.3936	$S_4 \succ S_2 \succ S_1 \succ S_3$
MOORA	0.3148	0.4485	0.1336	0.7251	$S_4 \succ S_2 \succ S_1 \succ S_3$
CRITIC	0.4023	0.5196	0.1938	0.9203	$S_4 \succ S_2 \succ S_1 \succ S_3$

10. Managerial implications

The results of this study have several practical implications for educational administrators and public-sector decision-makers responsible for allocating limited infrastructure funding. First, the proposed framework provides a transparent, auditable, and defensible basis for resource allocation. School 4 consistently emerged as the highest-priority alternative across the applied evaluation and ranking procedures, including the IFN-FWA-based assessment, TOPSIS, WASPAS, VIKOR, MOORA,

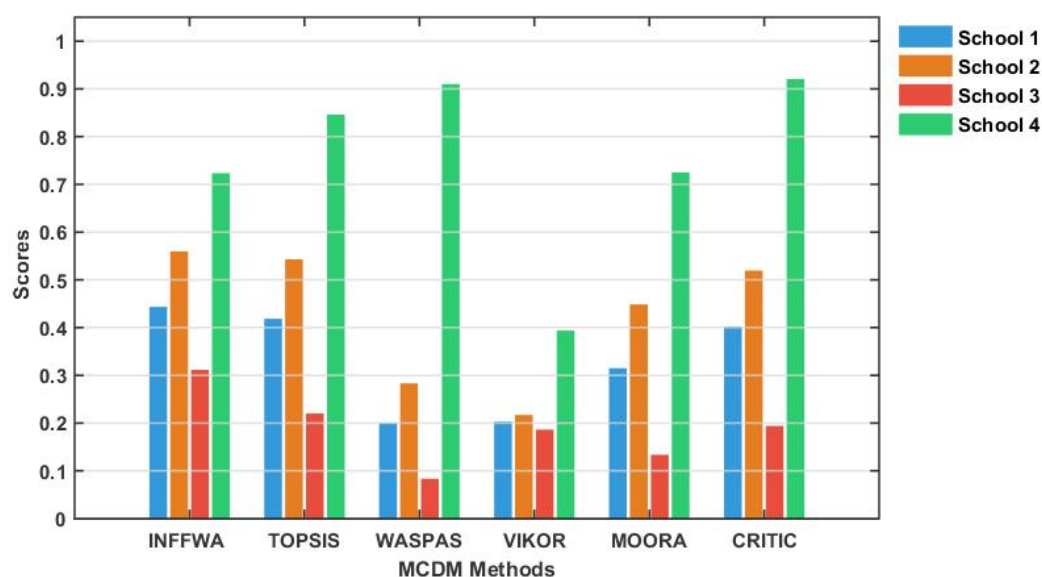


Fig. 8. Ranking of alternatives by IFFWA operator

and CRITIC-based analysis. The consistency of the ranking across different analytical approaches reduces dependence on the assumptions of any single method and provides decision-makers with stronger evidence when communicating and justifying investment priorities to school boards, parents, local authorities, and other community stakeholders. This is particularly important in public-sector decision-making, where infrastructure investments are subject to institutional scrutiny and require transparent justification.

Second, the framework provides a structured mechanism for handling qualitative uncertainty in criteria that cannot be represented adequately by precise numerical values. Criteria such as safety urgency and educational impact inherently involve expert judgment and may be characterized by incomplete information, hesitation, and different perceptions among evaluators. By representing these assessments through intuitionistic fuzzy numbers, the proposed framework explicitly incorporates both the degree of support for an assessment and the associated uncertainty. Consequently, experts are not required to express inherently imprecise judgments through artificially precise numerical scores, making the decision process more consistent with the conditions encountered in real-world infrastructure planning.

Another important practical implication concerns the determination of criterion weights. The use of the CRITIC method enables criterion importance to be derived objectively from the information contained in the decision matrix by considering both the variability of criterion values and the degree of conflict among criteria. This reduces reliance on administratively or politically negotiated weights and limits the influence of subjective preferences on the final prioritization. Such an approach can be particularly valuable in public resource-allocation problems in which stakeholders may have competing interests and different perceptions of criterion importance.

The proposed framework is also transferable to other public-investment decisions characterized by limited resources, competing priorities, and heterogeneous evaluation criteria. The same IFN-FWA/MCDM architecture can be adapted, for example, to the prioritization of healthcare infrastructure and medical equipment, road and public-transport maintenance projects, municipal facilities, or disaster-recovery investments. In such applications, decision-makers frequently need to consider safety, population affected, investment cost, urgency, and expected long-term benefits simultaneously, while also accounting for uncertainty in expert assessments.

Finally, the inclusion of VIKOR provides additional practical support when decision-makers need to consider compromise solutions rather than relying exclusively on the highest-ranked alternative. Its utility–regret structure and compromise-solution conditions can assist administrators in identifying acceptable alternatives when the initially preferred project cannot be implemented because of financial, regulatory, site-related, or other practical constraints. In such circumstances, the framework can support the selection of a defensible alternative without requiring the entire evaluation process to be repeated. Overall, these characteristics make the proposed approach suitable not only for identifying a preferred investment alternative but also for supporting transparent, adaptable, and accountable public-sector resource allocation.

11. Conclusion

This study developed an integrated intuitionistic fuzzy multi-criteria decision-making framework for prioritizing school infrastructure projects under limited financial resources. The proposed framework addresses a practical public-sector decision problem involving the selection of the highest-priority project among four school building alternatives considered for a one-time investment in classroom expansion. To represent the inherent uncertainty and hesitation associated with expert judgments, the evaluations of the alternatives were expressed using intuitionistic fuzzy numbers. Four decision criteria—safety urgency, number of students affected, cost, and educational impact—were considered, while the individual expert assessments were aggregated using the Frank Weighted Averaging operator. In this way, the proposed framework enables uncertainty in expert evaluations to be incorporated systematically into the decision-making process rather than being eliminated at an early stage of the analysis.

Following the aggregation process, the intuitionistic fuzzy assessments were transformed into a crisp decision matrix and analyzed using several MCDM techniques, including TOPSIS, WASPAS, VIKOR, and MOORA, together with CRITIC-based analysis. These methods rely on different mathematical principles and therefore provide complementary perspectives on the prioritization problem. Despite these methodological differences, the analysis produced a consistent final ordering of the alternatives. School 4 was identified as the highest-priority project, followed by School 2, School 1, and School 3. The stability of this ranking across the applied methods provides additional support for the robustness of the obtained results and indicates that the final recommendation is not dependent on the assumptions or computational mechanism of a single MCDM method. From a methodological perspective, the main contribution of the study lies in combining intuitionistic fuzzy information, Frank-based aggregation, and multi-method MCDM analysis within a unified decision-support framework. The intuitionistic fuzzy representation provides a suitable mechanism for modeling both membership and non-membership information in expert evaluations, while the Frank aggregation operator offers a flexible mechanism for synthesizing individual assessments. The subsequent application of multiple MCDM methods provides an additional validation layer by examining whether the prioritization remains stable under different ranking principles. This structure improves the transparency and methodological reliability of the decision process, which is particularly important in public-sector environments where resource allocation decisions must be clearly justified to multiple stakeholders.

From a practical perspective, the proposed framework provides decision makers with a systematic procedure for comparing infrastructure projects characterized by competing objectives and limited resources. Rather than relying exclusively on financial considerations or subjective judgments, the framework allows technical, social, educational, and economic criteria to be evaluated simultaneously while explicitly accounting for uncertainty in expert assessments. Consequently, the resulting prioritization can provide a more transparent and evidence-based foundation for allocating limited public resources.

Although the framework was demonstrated through the prioritization of school building projects,

its methodological structure is sufficiently general to be adapted to other public-sector resource allocation problems. Potential applications include healthcare infrastructure, transportation projects, municipal investments, public facility renovation, and other planning problems in which several alternatives must be evaluated against conflicting criteria under uncertain information. Future research could extend the proposed framework by considering larger sets of alternatives and criteria, alternative intuitionistic fuzzy aggregation operators, and additional objective or subjective criteria-weighting approaches. Further studies could also investigate the stability of the results through sensitivity and comparative analyses and examine the framework using larger-scale empirical applications involving different groups of stakeholders.

Conflict of Interest

The author declares no conflicts of interest.

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