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Fuzzy MultiLinear Sets, Neutrosophic MultiLinear Sets, abd Plithogenic MultiLinear Sets

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ABSTRACT

To address uncertainty, vagueness, and imprecision in decision-making, various extensions of classical set theory have been developed. Examples of such sets include fuzzy sets, neutrosophic sets, and plithogenic sets. A Multilinear Set consists of binary variables constrained by multilinear equations, where each auxiliary variable represents the product of selected primary binary variables. Extended concepts of the Multilinear Set using frameworks such as Fuzzy Sets have not yet been explored. To fill this gap, this paper investigates and analyzes the structures of Fuzzy Multilinear Sets, Neutrosophic Multilinear Sets, and Plithogenic Multilinear Sets.

1. Preliminaries

This section presents the fundamental concepts and definitions essential for the discussions in this paper. The study builds upon classical set theory and extends its principles into more advanced theoretical frameworks. For readers seeking a deeper understanding of foundational set theory, recommended references include [1]. Additionally, this paper exclusively considers finite, undirected, and simple sets (i.e., sets that are not multisets).

1.1 *Uncertain Sets*

A variety of set–theoretic extensions have been proposed to model imprecision, vagueness, and partial evidence in decision making. These frameworks enrich classical membership so that elements may belong to a set to nonbinary degrees and, in some cases, with explicitly modeled indeterminacy.

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One of the most widely used formalisms is the *fuzzy set* due to Zadeh [2, 3], in which each element of a universe is assigned a membership grade in $[0,1]$, thereby allowing partial inclusion. Atanassov's *intuitionistic fuzzy sets* extend this idea by recording, for every element, both a degree of membership and a degree of non-membership, subject to natural consistency constraints [4]. Smarandache's *neutrosophic sets* further separate the available information into three independent components—truth, indeterminacy, and falsity—each taking values in $[0,1]$ [5–7]. Related concepts include Complex Neutrosophic Sets[8–10], Bipolar Neutrosophic Sets[11, 12], Interval-Valued Neutrosophic Sets[13, 14], and Pythagorean Neutrosophic Sets[15, 16]. Higher-order variants such as *hyperneutrosophic sets* have also been studied to capture layered uncertainties [17, 18]. More recently, *plithogenic sets* have been introduced to model attribute-driven membership together with an explicit *degree-of-contradiction* between attribute values, offering a fine-grained account of multi-criteria and internally conflicting information [19–22].

Definition 1.1 (Set). *[1] A* set *is a well-specified collection of objects, called* elements*. If an object x belongs to a set A, we write $x \in A$. Sets are typically displayed by listing their elements within curly braces.*

Definition 1.2 (Subset). *[1] For sets A and B, we say that A is a* subset *of B, written $A \subseteq B$, if every element of A is also an element of B:*

$$\forall x \left(x \in A \Rightarrow x \in B\right).$$

If $A \subseteq B$ and $A \neq B$, then A is a proper subset *of B, denoted $A \subset B$.*

Definition 1.3 (Empty Set). *[1] The* empty set *$\emptyset$ is the unique set with no elements: $\forall x\,(x \notin \emptyset)$. Moreover, $\emptyset$ is a subset of every set A, i.e., $\emptyset \subseteq A$.*

Definition 1.4 (Fuzzy Set and Fuzzy Relation). *[2, 23] Let Y be a nonempty universe. A* fuzzy set *on Y is a map $\tau : Y \to [0,1]$; the value $\tau(y)$ is the membership grade of $y \in Y$. A* fuzzy relation *on Y is a function $\delta : Y \times Y \to [0,1]$. We call δ a fuzzy relation* on *τ if, for all $y, z \in Y$,*

$$\delta(y,z) \ \leq \ \min\{\tau(y),\, \tau(z)\}.$$

Example 1.5 (Fuzzy set & fuzzy relation: coffee–blend suitability). *Let $Y = \{\mathrm{Kenya}, \mathrm{Brazil}, \mathrm{Sumatra}\}$ be three single-origin coffees. Define the fuzzy set* "espresso-suitable" *by the membership map*

$$\tau(\mathrm{Kenya}) = 0.80, \qquad \tau(\mathrm{Brazil}) = 0.60, \qquad \tau(\mathrm{Sumatra}) = 0.40.$$

Interpret $\tau(y)$ as how suitable origin y is (on $[0,1]$) for espresso on its own.

Define a fuzzy relation $\delta : Y \times Y \to [0,1]$ for "blend is espresso-suitable" *by*

$$\delta(y,z) \ := \ \min\{\tau(y),\, \tau(z)\}.$$

Then, for instance, $\delta(\mathrm{Kenya},\mathrm{Brazil}) = \min\{0.80, 0.60\} = 0.60$ and $\delta(\mathrm{Brazil},\mathrm{Sumatra}) = \min\{0.60, 0.40\} = 0.40$. By construction $\delta(y,z) \leq \min\{\tau(y), \tau(z)\}$ for all $y, z \in Y$, so δ is a fuzzy relation on τ.

Definition 1.6 (Neutrosophic Set). *[5, 24] Let X be a nonempty set. A* neutrosophic set *A on X is specified by three functions*

$$T_A,\ I_A,\ F_A : X \to [0,1],$$

where, for each $x \in X$, $T_A(x)$, $I_A(x)$, and $F_A(x)$ denote the degrees of truth, indeterminacy, and falsity, respectively, with

$$0 \ \leq \ T_A(x) + I_A(x) + F_A(x) \ \leq \ 3.$$

Example 1.7 (Neutrosophic set: spam assessment for incoming emails). *Let $X = \{e_1, e_2, e_3\}$ be three emails arriving to an inbox. Consider the neutrosophic set A = "is spam", encoded by the three functions $T_A, I_A, F_A : X \to [0, 1]$ (truth/indeterminacy/falsity degrees):*

$$(T_A(e_1), I_A(e_1), F_A(e_1)) = (0.85, 0.05, 0.20),$$
$$(T_A(e_2), I_A(e_2), F_A(e_2)) = (0.40, 0.40, 0.50),$$
$$(T_A(e_3), I_A(e_3), F_A(e_3)) = (0.10, 0.15, 0.90).$$

Here e_1 is very likely spam with little uncertainty, e_2 is ambiguous (large indeterminacy), and e_3 is very likely not spam. Each triple lies in $[0, 1]^3$ and satisfies $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

Definition 1.8 (Plithogenic Set). *[19, 22] Let S be a universe and $P \subseteq S$ a nonempty subset. A* plithogenic set *is a tuple*

$$PS = (P,\ v,\ Pv,\ \mathrm{pdf},\ pCF),$$

consisting of:

- *an* attribute v*;*
- *its* value domain Pv*;*
- *a* degree-of-appurtenance function $\mathrm{pdf} : P \times Pv \to [0, 1]^s$ *assigning (possibly vector-valued) membership degrees to element–value pairs;*
- *a* degree-of-contradiction function $pCF : Pv \times Pv \to [0, 1]^t$ *quantifying the contradiction between attribute values.*

The contradiction function satisfies, for all $a, b \in Pv$,

$$pCF(a, a) = 0 \quad \textit{(reflexivity)}, \qquad pCF(a, b) = pCF(b, a) \quad \textit{(symmetry)}.$$

Example 1.9 (Plithogenic set: jacket selection by season with contradiction between seasons). *Let S be a catalog of jackets and $P = \{J_{\text{wool}}, J_{\text{linen}}, J_{\text{soft}}\} \subseteq S$ the candidate items. Take the attribute v = "season" with value domain $Pv = \{\text{Winter}, \text{Summer}, \text{AllSeason}\}$. Define the degree-of-appurtenance function $\mathrm{pdf} : P \times Pv \to [0, 1]$ (scalar case $s = 1$) by*

	Winter	Summer	AllSeason
J_{wool}	0.95	0.05	0.60
J_{linen}	0.10	0.90	0.55
J_{soft}	0.70	0.40	0.80

and the degree-of-contradiction function $pCF : Pv \times Pv \to [0, 1]$ (scalar case $t = 1$) by the symmetric matrix

pCF	Winter	Summer	AllSeason
Winter	0	1.0	0.4
Summer	1.0	0	0.4
AllSeason	0.4	0.4	0

which satisfies reflexivity $pCF(a, a) = 0$ and symmetry $pCF(a, b) = pCF(b, a)$. Thus

$$PS = (P,\ v,\ Pv,\ \mathrm{pdf},\ pCF)$$

is a plithogenic set: membership depends on (item, season) pairs, while contradictions between seasons (e.g., Winter vs. Summer is 1.0) modulate multi-attribute aggregation.

1.2 Multilinear Set

We define the Multilinear Set. A Multilinear Set is a collection of binary variables constrained by multilinear equations, where each auxiliary variable represents the product of selected primary binary variables [25–28].

Definition 1.10 (Index Set)**.** *(cf.[25]) An* index set *is a set whose elements are used to label or index the members of another collection. For example,*

$$N = \{1, 2, \dots, n\}$$

is a common index set.

Definition 1.11 (Product)**.** *The* product *of a finite sequence $a_1, a_2, \dots, a_k$ is defined as*

$$\prod_{i=1}^{k} a_i = a_1 \cdot a_2 \cdots a_k.$$

Definition 1.12 (Convex Hull)**.** *(cf.[29–31]) The* convex hull *of a set S in a vector space is the smallest convex set that contains S, defined by*

$$\operatorname{conv}(S) = \left\{ \sum_{i=1}^{k} \lambda_i x_i : x_i \in S,\ \lambda_i \geq 0,\ \sum_{i=1}^{k} \lambda_i = 1,\ k \in \mathbb{N} \right\}.$$

Definition 1.13 (Multilinear Set)**.** *[25, 32] Let $N = \{1, 2, \dots, n\}$ be an index set and let $\mathcal{I}$ be a family of subsets of N such that for every $I \in \mathcal{I}$, $|I| \geq 2$. The* Multilinear Set *S is defined as*

$$S = \Big\{ (x, y) \in \{0,1\}^{n+|\mathcal{I}|} \ : \ y_I = \prod_{i \in I} x_i, \quad \forall\, I \in \mathcal{I} \Big\},$$

where $x = (x_1, x_2, \dots, x_n) \in \{0,1\}^n$ represents binary decision variables and, for each $I \in \mathcal{I}$, y_I is a binary variable equal to the product $\prod_{i \in I} x_i$. The convex hull of S is known as the Multilinear Polytope.

Example 1.14 (Simple Multilinear Set)**.** *Consider the case $n = 3$ with the family $\mathcal{I} = \{\{1,2\}, \{1,3\}, \{2,3\}\}$. Then the Multilinear Set S is given by*

$$S = \left\{ (x_1, x_2, x_3, y_{12}, y_{13}, y_{23}) \in \{0,1\}^6 \ : \ \begin{array}{rcl} y_{12} & = & x_1 \cdot x_2, \\ y_{13} & = & x_1 \cdot x_3, \\ y_{23} & = & x_2 \cdot x_3, \end{array} \right\}.$$

For example, if $(x_1, x_2, x_3) = (1, 0, 1)$, then

$$y_{12} = 1 \cdot 0 = 0, \quad y_{13} = 1 \cdot 1 = 1, \quad y_{23} = 0 \cdot 1 = 0.$$

Thus, one element of S is $(1, 0, 1, 0, 1, 0)$.

2. Results of This Paper

This section presents the main findings discussed in this paper.

2.1 Fuzzy Multilinear Set

We examine the Fuzzy Multilinear Set. A fuzzy multilinear set assigns membership degrees to variable products under multilinear constraints, measuring closeness between observed and ideal products.

Definition 2.1 (Fuzzy Multilinear Set)**.** *Let $n \in \mathbb{N}$ and let*

$$\mathcal{I} \subseteq 2^{\{1,2,\ldots,n\}}$$

be a family of subsets satisfying $|I| \geq 2$ for every $I \in \mathcal{I}$. For each $I \in \mathcal{I}$, define the classical multilinear function

$$t_I(x) = \prod_{i \in I} x_i, \quad \text{for } x = (x_1, x_2, \ldots, x_n) \in \{0,1\}^n.$$

For a candidate vector $y = (y_I)_{I \in \mathcal{I}} \in [0,1]^{|\mathcal{I}|}$, define for each $I \in \mathcal{I}$ the satisfaction measure

$$f_I(x, y_I) = 1 - |y_I - t_I(x)|\,.$$

The overall membership function is then given by

$$\mu(x, y) = \min_{I \in \mathcal{I}} f_I(x, y_I).$$

The Fuzzy Multilinear Set *is the fuzzy set*

$$\tilde{S}_F = \Big\{ \big((x, y), \mu(x, y)\big) : x \in \{0,1\}^n,\ y \in [0,1]^{|\mathcal{I}|} \Big\}.$$

Example 2.2 (Fuzzy Multilinear Set Example)**.** *Consider $n = 3$ and let $\mathcal{I} = \{\{1,2\}, \{1,3\}, \{2,3\}\}$. Then the classical multilinear functions are:*

$$t_{\{1,2\}}(x) = x_1 x_2, \quad t_{\{1,3\}}(x) = x_1 x_3, \quad t_{\{2,3\}}(x) = x_2 x_3.$$

Take $x = (1, 0, 1) \in \{0,1\}^3$ so that

$$t_{\{1,2\}}(x) = 0, \quad t_{\{1,3\}}(x) = 1, \quad t_{\{2,3\}}(x) = 0.$$

Choose $y = (y_{12}, y_{13}, y_{23}) = (0.1,\ 0.9,\ 0.0) \in [0,1]^3$. Then:

$$f_{\{1,2\}}(x, y_{12}) = 1-|0.1-0| = 0.9, \quad f_{\{1,3\}}(x, y_{13}) = 1-|0.9-1| = 0.9, \quad f_{\{2,3\}}(x, y_{23}) = 1-|0-0| = 1.$$

Thus, the overall membership is

$$\mu(x, y) = \min\{0.9,\ 0.9,\ 1\} = 0.9.$$

Hence, the pair $\big((1,0,1), (0.1, 0.9, 0.0)\big)$ belongs to $\tilde{S}_F$ with membership degree 0.9.

Example 2.3 (Fuzzy Multilinear Set in Project Synergy Assessment)**.** *Synergy refers to the enhanced effect achieved when multiple elements interact, producing a combined outcome greater than their individual contributions (cf.[33, 34]). Consider a scenario in which a company must decide whether to invest in three projects, labeled A, B, and C. Let*

$$x = (x_1, x_2, x_3) \in \{0,1\}^3$$

denote the binary investment decisions for projects A, B, and C respectively (with $x_i = 1$ indicating an investment). Synergy [35] effects between projects are captured by auxiliary variables y_I for each pair I in

$$\mathcal{I} = \{\{1,2\},\{1,3\},\{2,3\}\}.$$

For each $I \in \mathcal{I}$, the ideal (classical) synergy is given by

$$t_I(x) = \prod_{i \in I} x_i.$$

Suppose the company decides to invest in projects A and C but not in B, so that

$$x = (1,0,1).$$

Then, the ideal synergy values are:

$$t_{\{1,2\}}(x) = 1 \cdot 0 = 0, \quad t_{\{1,3\}}(x) = 1 \cdot 1 = 1, \quad t_{\{2,3\}}(x) = 0 \cdot 1 = 0.$$

Due to uncertainties in measuring synergy effects, the observed synergy degrees are fuzzy. Assume the observed vector is

$$y = (y_{12}, y_{13}, y_{23}) = (0.2,\ 0.8,\ 0.1),$$

with each $y_I \in [0,1]$ representing the extent to which the expected synergy is realized. Define the satisfaction measure for each I as

$$f_I(x, y_I) = 1 - \big|y_I - t_I(x)\big|.$$

Then, we obtain:

$$f_{\{1,2\}}(x, y_{12}) = 1 - |0.2 - 0| = 0.8,$$
$$f_{\{1,3\}}(x, y_{13}) = 1 - |0.8 - 1| = 0.8,$$
$$f_{\{2,3\}}(x, y_{23}) = 1 - |0.1 - 0| = 0.9.$$

The overall membership function is defined as

$$\mu(x,y) = \min\{0.8,\ 0.8,\ 0.9\} = 0.8.$$

Thus, the fuzzy multilinear set

$$\tilde{S}_F = \Big\{ \big((x,y), \mu(x,y)\big) : x \in \{0,1\}^3,\ y \in [0,1]^3 \Big\}$$

assigns the pair $((1,0,1),(0.2,0.8,0.1))$ a membership degree of 0.8, indicating that the project synergy is achieved to a high (80%) but not perfect degree.

Theorem 2.4 (Support Theorem for Fuzzy Multilinear Set). *A point $(x,y) \in \{0,1\}^n \times [0,1]^{|\mathcal{I}|}$ satisfies $\mu(x,y) = 1$ if and only if*

$$y_I = t_I(x) \quad \text{for all } I \in \mathcal{I}.$$

That is, the support of $\tilde{S}_F$ coincides with the classical multilinear set

$$S = \Big\{ (x,y) \in \{0,1\}^{n+|\mathcal{I}|} : y_I = \prod_{i \in I} x_i, \quad \forall\, I \in \mathcal{I} \Big\}.$$

Proof. For any $I \in \mathcal{I}$, observe that

$$f_I(x, y_I) = 1 - |y_I - t_I(x)|.$$

Thus, $f_I(x, y_I) = 1$ if and only if $|y_I - t_I(x)| = 0$, which is equivalent to $y_I = t_I(x)$. Since $\mu(x,y) = \min_{I \in \mathcal{I}} f_I(x, y_I)$, we have $\mu(x,y) = 1$ if and only if $f_I(x, y_I) = 1$ for all $I \in \mathcal{I}$. Therefore, $\mu(x,y) = 1$ if and only if $y_I = t_I(x)$ for every $I \in \mathcal{I}$. □

2.2 Neutrosophic Multilinear Set

We examine the Neutrosophic Multilinear Set. A neutrosophic multilinear set assigns truth, indeterminacy, and falsity degrees to variable products, aggregating deviations from ideal multilinear relationships globally.

Definition 2.5 (Neutrosophic Multilinear Set)**.** *Let $n \in \mathbb{N}$ and let $\mathcal{I}$ be a family of subsets of $\{1, 2, \ldots, n\}$ with $|I| \geq 2$ for each $I \in \mathcal{I}$. For $x \in \{0,1\}^n$ and $y \in [0,1]^{|\mathcal{I}|}$, define for each $I \in \mathcal{I}$:*

$$T_I(x, y_I) = 1 - |y_I - t_I(x)|, \quad I_I(x, y_I) = |y_I - t_I(x)|, \quad F_I(x, y_I) = 0.$$

Aggregate these measures by setting

$$T(x, y) = \min_{I \in \mathcal{I}} T_I(x, y_I), \quad I(x, y) = \max_{I \in \mathcal{I}} I_I(x, y_I),$$

and define the falsity component by

$$F(x, y) = 1 - T(x, y) - I(x, y).$$

The Neutrosophic Multilinear Set *is then given by the mapping*

$$\nu : \{0,1\}^n \times [0,1]^{|\mathcal{I}|} \to [0,1]^3, \quad \nu(x, y) = \big(T(x, y),\ I(x, y),\ F(x, y)\big).$$

Example 2.6 (Neutrosophic Multilinear Set Example)**.** *With $n = 3$ and $\mathcal{I} = \{\{1,2\}, \{1,3\}, \{2,3\}\}$, let $x = (1, 0, 1)$ so that*

$$t_{\{1,2\}}(x) = 0, \quad t_{\{1,3\}}(x) = 1, \quad t_{\{2,3\}}(x) = 0.$$

Choose $y = (0.1,\ 0.9,\ 0.0)$. Then, for each I:

$$T_{\{1,2\}}(x, y_{12}) = 1 - |0.1 - 0| = 0.9, \quad I_{\{1,2\}}(x, y_{12}) = 0.1;$$

$$T_{\{1,3\}}(x, y_{13}) = 1 - |0.9 - 1| = 0.9, \quad I_{\{1,3\}}(x, y_{13}) = 0.1;$$

$$T_{\{2,3\}}(x, y_{23}) = 1 - |0 - 0| = 1, \quad I_{\{2,3\}}(x, y_{23}) = 0.$$

Thus,

$$T(x, y) = \min\{0.9,\ 0.9,\ 1\} = 0.9, \quad I(x, y) = \max\{0.1,\ 0.1,\ 0\} = 0.1,$$

and

$$F(x, y) = 1 - 0.9 - 0.1 = 0.$$

Therefore, $\nu((1,0,1), (0.1, 0.9, 0.0)) = (0.9, 0.1, 0)$.

Example 2.7 (Neutrosophic Multilinear Set in Quality Control Assessment)**.** *Consider a quality control [36–38] process in which a product is subjected to three independent tests. Let*

$$x = (x_1, x_2, x_3) \in \{0,1\}^3$$

denote the outcomes of these tests (with $x_i = 1$ indicating a pass and $x_i = 0$ indicating a failure). Pairs of tests jointly determine an overall quality indicator via auxiliary variables y_I for each I in

$$\mathcal{I} = \{\{1,2\}, \{1,3\}, \{2,3\}\},$$

with the ideal relation

$$t_I(x) = \prod_{i \in I} x_i.$$

Assume the product passes tests 1 and 2 but fails test 3, so that

$$x = (1, 1, 0).$$

Then, the ideal outcomes are:

$$t_{\{1,2\}}(x) = 1 \cdot 1 = 1, \quad t_{\{1,3\}}(x) = 1 \cdot 0 = 0, \quad t_{\{2,3\}}(x) = 1 \cdot 0 = 0.$$

Due to measurement uncertainties, the observed values are given by

$$y = (y_{12}, y_{13}, y_{23}) = (0.9,\ 0.2,\ 0.3).$$

For each I, define the truth and indeterminacy measures as

$$T_I(x, y_I) = 1 - |y_I - t_I(x)|, \quad I_I(x, y_I) = |y_I - t_I(x)|,$$

and set $F_I(x, y_I) = 0$. Then:

$$T_{\{1,2\}}(x, y_{12}) = 1 - |0.9 - 1| = 0.9, \quad I_{\{1,2\}}(x, y_{12}) = 0.1;$$

$$T_{\{1,3\}}(x, y_{13}) = 1 - |0.2 - 0| = 0.8, \quad I_{\{1,3\}}(x, y_{13}) = 0.2;$$

$$T_{\{2,3\}}(x, y_{23}) = 1 - |0.3 - 0| = 0.7, \quad I_{\{2,3\}}(x, y_{23}) = 0.3.$$

Aggregate these by defining

$$T(x, y) = \min\{0.9,\ 0.8,\ 0.7\} = 0.7, \quad I(x, y) = \max\{0.1,\ 0.2,\ 0.3\} = 0.3,$$

and setting

$$F(x, y) = 1 - T(x, y) - I(x, y) = 1 - 0.7 - 0.3 = 0.$$

Thus, the neutrosophic mapping

$$\nu(x, y) = \big(T(x, y),\ I(x, y),\ F(x, y)\big)$$

assigns the pair $((1, 1, 0), (0.9, 0.2, 0.3))$ the value $(0.7, 0.3, 0)$, indicating that the product meets the quality standard with a truth degree of 70% and an indeterminacy of 30% (with no falsity). This reflects the inherent uncertainty in the measurement process.

Theorem 2.8 (Generalization of Neutrosophic Sets and Fuzzy Multilinear Sets)**.** *Let*

$$\nu(x, y) = \big(T(x, y),\ I(x, y),\ F(x, y)\big)$$

be the neutrosophic membership mapping for the Neutrosophic Multilinear Set, defined via

$$T(x, y) = \min_{I \in \mathcal{I}} \Big\{1 - |y_I - t_I(x)|\Big\}, \quad I(x, y) = \max_{I \in \mathcal{I}} \Big\{|y_I - t_I(x)|\Big\},$$

and

$$F(x, y) = 1 - T(x, y) - I(x, y).$$

Then:

1. $\nu(x, y) = (1, 0, 0)$ *if and only if* $y_I = t_I(x)$ *for all* $I \in \mathcal{I}$, i.e. $(x, y) \in S$.
2. *In this manner, the Neutrosophic Multilinear Set reduces to the classical Multilinear Set when indeterminacy and falsity vanish, thus generalizing both neutrosophic sets and Fuzzy Multilinear Sets.*

Proof. Assume that for every $I \in \mathcal{I}$, $y_I = t_I(x)$. Then for each I,

$$T_I(x, y_I) = 1 - |y_I - t_I(x)| = 1 \quad \text{and} \quad I_I(x, y_I) = |y_I - t_I(x)| = 0.$$

Hence, $T(x, y) = 1$ and $I(x, y) = 0$, which implies $F(x, y) = 1 - 1 - 0 = 0$; that is, $\nu(x, y) = (1, 0, 0)$. Conversely, if $\nu(x, y) = (1, 0, 0)$, then $T(x, y) = 1$ and $I(x, y) = 0$; hence, for every $I \in \mathcal{I}$, $T_I(x, y_I) = 1$ and so $|y_I - t_I(x)| = 0$. Therefore, $y_I = t_I(x)$ for all $I \in \mathcal{I}$, meaning $(x, y) \in S$. This proves the theorem. □

Theorem 2.9 (Support Theorem for Neutrosophic Multilinear Set). *A point* $(x, y) \in \{0, 1\}^n \times [0, 1]^{|\mathcal{I}|}$ *satisfies*

$$\nu(x, y) = (1, 0, 0)$$

if and only if

$$y_I = t_I(x) \quad \text{for all } I \in \mathcal{I}.$$

That is, the set of points with full truth-membership, zero indeterminacy, and zero falsity corresponds exactly to the classical multilinear set.

Proof. Suppose that for all $I \in \mathcal{I}$, $y_I = t_I(x)$. Then for each I,

$$T_I(x, y_I) = 1 - |y_I - t_I(x)| = 1, \quad I_I(x, y_I) = |y_I - t_I(x)| = 0.$$

Thus, $T(x, y) = \min_{I \in \mathcal{I}} 1 = 1$ and $I(x, y) = \max_{I \in \mathcal{I}} 0 = 0$. It follows that

$$F(x, y) = 1 - 1 - 0 = 0.$$

Conversely, if $\nu(x, y) = (1, 0, 0)$, then $T(x, y) = 1$ and $I(x, y) = 0$. In particular, for every $I \in \mathcal{I}$,

$$T_I(x, y_I) = 1 - |y_I - t_I(x)| = 1,$$

which implies $|y_I - t_I(x)| = 0$ and hence $y_I = t_I(x)$ for all I. □

2.3 Plithogenic Multilinear Set

We examine the Plithogenic Multilinear Set. A plithogenic multilinear set couples multilinear constraints with attribute-based membership and contradiction functions, representing context-dependent adherence of variable product configurations.

Definition 2.10 (Plithogenic Multilinear Set). *Let*

$$S = \Big\{(x, y) \in \{0, 1\}^{n+|\mathcal{I}|} : y_I = \prod_{i \in I} x_i, \quad \forall I \in \mathcal{I}\Big\}$$

be the classical multilinear set. Let v *be an attribute associated with each element* (x, y) *taking values in the set* Pv*. Define a* Degree of Appurtenance Function

$$\tilde{pdf} : S \times Pv \to [0, 1]^s,$$

which, for each $(x, y) \in S$ and for a given attribute value $a \in Pv$, quantifies the degree to which (x, y) adheres to the multilinear constraints under a. In addition, define the Degree of Contradiction Function

$$pCF : Pv \times Pv \to [0, 1]^t,$$

which measures the level of contradiction between pairs of attribute values, and satisfies:

$$pCF(a, a) = 0 \quad \textit{and} \quad pCF(a, b) = pCF(b, a), \quad \forall\, a, b \in Pv.$$

The Plithogenic Multilinear Set *is then defined as the tuple*

$$PS = \Big(S,\, v,\, Pv,\, \tilde{pdf},\, pCF\Big).$$

Example 2.11 (Plithogenic Multilinear Set Example)**.** *Consider $n = 3$ with $\mathcal{I} = \{\{1, 2\}, \{1, 3\}, \{2, 3\}\}$ so that*

$$S = \Big\{(x, y) \in \{0, 1\}^6 : y_{12} = x_1x_2,\ y_{13} = x_1x_3,\ y_{23} = x_2x_3\Big\}.$$

Let the attribute v denote "measurement reliability" with

$$Pv = \{\text{High},\ \text{Medium},\ \text{Low}\}.$$

For each $(x, y) \in S$, the function $\tilde{pdf}(x, y)$ returns a vector in $[0, 1]^s$ representing the degree to which the point adheres to the ideal multilinear relations under the given reliability level. For example, assume that when the attribute value is High, a point that perfectly satisfies the multilinear equations receives $\tilde{pdf}(x, y) = (1, 1, \ldots, 1)$. For a specific point, take

$$(x, y) = (1, 0, 1, 0, 1, 0) \in S,$$

and suppose the assigned attribute is High so that

$$\tilde{pdf}((1, 0, 1, 0, 1, 0), \text{High}) = (1, 1, \ldots, 1).$$

Moreover, the contradiction function pCF might be defined as, for instance,

$$pCF(\text{High}, \text{Medium}) = 0.3, \quad pCF(\text{High}, \text{Low}) = 0.8, \quad pCF(\text{Medium}, \text{Low}) = 0.5,$$

with the property $pCF(a, a) = 0$ for all $a \in Pv$.

Thus, the Plithogenic Multilinear Set is

$$PS = \Big(S,\, v,\, Pv,\, \tilde{pdf},\, pCF\Big),$$

which integrates the classical multilinear structure with additional information on uncertainty and attribute contradictions.

Theorem 2.12 (Generalization of Plithogenic Sets and Neutrosophic Multilinear Sets)**.** *Let*

$$PS = \Big(S,\, v,\, Pv,\, \tilde{pdf},\, pCF\Big)$$

be a Plithogenic Multilinear Set, where S is the classical Multilinear Set defined by

$$S = \Big\{(x, y) \in \{0, 1\}^{n+|\mathcal{I}|} : y_I = \prod_{i \in I} x_i,\ \forall\, I \in \mathcal{I}\Big\},$$

v is an attribute with range Pv, $\tilde{pdf} : S \times Pv \to [0, 1]^s$ is the Degree of Appurtenance Function, and $pCF : Pv \times Pv \to [0, 1]^t$ is the Degree of Contradiction Function satisfying $pCF(a, a) = 0$ and symmetry. Then, for any fixed attribute value $a \in Pv$ satisfying $pCF(a, a) = 0$:

1. *If* $\tilde{pdf}(x, y, a) = (1, 1, \ldots, 1)$ *for* $(x, y) \in S$*, then* $y_I = \prod_{i \in I} x_i$ *for all* $I \in \mathcal{I}$*; that is,* (x, y) *satisfies the multilinear constraints exactly.*
2. *Conversely, if* $(x, y) \in S$*, then it is possible to assign an attribute value* $a \in Pv$ *for which* $\tilde{pdf}(x, y, a) = (1, 1, \ldots, 1)$.

Thus, the Plithogenic Multilinear Set generalizes both the Plithogenic Set and the Neutrosophic Multilinear Set.

Proof. Assume that for a fixed $a \in Pv$ (with $pCF(a, a) = 0$), we have

$$\tilde{pdf}(x, y, a) = (1, 1, \ldots, 1)$$

for a given $(x, y) \in S$. By the definition of $\tilde{pdf}$, this maximal membership vector is achieved if and only if, for every $I \in \mathcal{I}$,

$$\tilde{pdf}_I(x, y, a) = 1 \quad \Longleftrightarrow \quad |y_I - t_I(x)| = 0.$$

Thus, for every $I \in \mathcal{I}$,

$$y_I = t_I(x) = \prod_{i \in I} x_i.$$

Conversely, if $(x, y) \in S$ then by definition $y_I = \prod_{i \in I} x_i$ for all I, so one can define the Degree of Appurtenance Function such that $\tilde{pdf}(x, y, a) = (1, 1, \ldots, 1)$ for a suitably chosen attribute value a (with $pCF(a, a) = 0$). This demonstrates that the plithogenic framework encompasses the classical structure when the membership is maximal and no contradiction is present. □

Theorem 2.13 (Support Theorem for Plithogenic Multilinear Set). *Let* $PS = \left(S,\, v,\, Pv,\, \tilde{pdf},\, pCF\right)$ *be a Plithogenic Multilinear Set. If for some* $a \in Pv$ *with* $pCF(a, a) = 0$*, an element* $(x, y) \in S$ *satisfies*

$$\tilde{pdf}(x, y, a) = (1, 1, \ldots, 1),$$

then

$$y_I = \prod_{i \in I} x_i \quad \textit{for all } I \in \mathcal{I},$$

i.e., (x, y) *belongs to the classical multilinear set.*

Proof. Assume that for a fixed attribute value $a \in Pv$ (with $pCF(a, a) = 0$), we have

$$\tilde{pdf}(x, y, a) = (1, 1, \ldots, 1)$$

for a given $(x, y) \in S$. By the definition of the degree of appurtenance function $\tilde{pdf}$, this maximal membership vector is achieved if and only if the deviation from the ideal multilinear relationship is zero. That is, for every $I \in \mathcal{I}$,

$$\tilde{pdf}_I(x, y, a) = 1 \quad \Longleftrightarrow \quad |y_I - t_I(x)| = 0.$$

Hence, for each $I \in \mathcal{I}$,

$$|y_I - \prod_{i \in I} x_i| = 0 \quad \Longrightarrow \quad y_I = \prod_{i \in I} x_i.$$

Since this holds for every $I \in \mathcal{I}$, the point (x, y) satisfies all the multilinear constraints exactly. Moreover, because the contradiction function pCF satisfies $pCF(a, a) = 0$, there is no contradiction penalty when the attribute value is a. Therefore, the condition $\tilde{pdf}(x, y, a) = (1, 1, \ldots, 1)$ implies that (x, y) belongs to the classical multilinear set

$$S = \Big\{ (x, y) \in \{0, 1\}^{n + |\mathcal{I}|} : y_I = \prod_{i \in I} x_i,\ \forall\, I \in \mathcal{I} \Big\}.$$

This completes the proof. □

2.4 Multilinear Graph

In analogy with multilinear sets, we extend the concept to graph theory by incorporating binary variables for both vertices and edges, along with multilinear constraints that capture the incidence relationships.

Definition 2.14 (Graph). *[39] A graph G is a mathematical structure consisting of a set of vertices $V(G)$ and a set of edges $E(G)$ that connect pairs of vertices, representing relationships or connections between them. Formally, a graph is defined as $G = (V, E)$, where V is the vertex set and E is the edge set.*

Definition 2.15 (Multilinear Graph). *Let $G = (V, E)$ be a finite, simple, undirected graph, where*

$$V = \{v_1, v_2, \ldots, v_n\}$$

is the vertex set and

$$E \subseteq \Big\{\{u, v\} \mid u, v \in V,\ u \neq v\Big\}$$

is the edge set. Associate with each vertex $v \in V$ a binary variable $x_v \in \{0, 1\}$ and with each edge $e = \{u, v\} \in E$ a binary variable $y_e \in \{0, 1\}$. The Multilinear Graph *of G is defined as the set*

$$MG(G) = \Big\{(x, y) \in \{0, 1\}^{|V|+|E|} : \forall\, e = \{u, v\} \in E, \quad y_e = x_u \cdot x_v\Big\}.$$

Example 2.16 (Multilinear Graph). *Consider the graph*

$$G = (V, E), \quad \text{with} \quad V = \{A, B, C\} \quad \text{and} \quad E = \{\{A, B\}, \{B, C\}\}.$$

Associate binary variables x_A, x_B, x_C with vertices A, B, C and variables y_{AB} and y_{BC} with edges $\{A, B\}$ and $\{B, C\}$, respectively. Then the multilinear graph is defined as

$$MG(G) = \Big\{(x_A, x_B, x_C, y_{AB}, y_{BC}) \in \{0, 1\}^5 : y_{AB} = x_A \cdot x_B,\ y_{BC} = x_B \cdot x_C\Big\}.$$

For instance, if

$$(x_A, x_B, x_C) = (1, 0, 1),$$

then

$$y_{AB} = 1 \cdot 0 = 0 \quad \text{and} \quad y_{BC} = 0 \cdot 1 = 0.$$

Thus, one element of $MG(G)$ is $(1, 0, 1, 0, 0)$.

Theorem 2.17 (Projection Theorem). *For the multilinear graph $MG(G)$, the projection onto the vertex variables is the entire binary space:*

$$\pi_V\big(MG(G)\big) = \{0, 1\}^{|V|}.$$

Proof. Let $x \in \{0, 1\}^{|V|}$ be arbitrary. For each edge $e = \{u, v\} \in E$, define $y_e = x_u \cdot x_v$. Since $x_u, x_v \in \{0, 1\}$, it follows that $y_e \in \{0, 1\}$. Therefore, the vector $(x, y) \in MG(G)$ and the projection of (x, y) onto the vertex variables is exactly x. Hence, every binary vector x appears in the projection. □

Theorem 2.18 (Uniqueness of Edge Variables). *For a given vertex assignment $x \in \{0, 1\}^{|V|}$, there exists a unique corresponding vector $y \in \{0, 1\}^{|E|}$ such that $(x, y) \in MG(G)$.*

Proof. Let $x \in \{0, 1\}^{|V|}$ be given. For each edge $e = \{u, v\} \in E$, define $y_e = x_u \cdot x_v$. Since the product of two binary values is uniquely determined in $\{0, 1\}$, there is exactly one choice for y_e corresponding to the given x. Hence, the mapping

$$x \mapsto (x, y), \quad \text{with } y_e = x_u \cdot x_v \text{ for all } e \in E,$$

is well-defined and unique. This proves the theorem. □

3. Conclusion

This paper has examined and analyzed the structural characteristics of Fuzzy Multilinear Sets, Neutrosophic Multilinear Sets, and Plithogenic Multilinear Sets.

In future research, it is expected that further investigations will explore the extensions of Fuzzy Multilinear Sets, Neutrosophic Multilinear Sets, and Plithogenic Multilinear Sets by employing mathematical frameworks such as HyperGraphs[40, 41], HyperAlgebras[42], SuperHyperGraphs[43–47], and HyperStructures[48–50].

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Conflicts of Interest
The authors declare no conflicts of interest.

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