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Solution of an Ameliorating Inventory Model with Ramp-type of Demand and Inflation: A Fuzzy Optimization Approach

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ABSTRACT

In this article, ameliorating items whose quality improves during the early stages of an inventory cycle are considered. The demand rate for ameliorating items is generally unstable. There are times when it rises, periods when it stays the same, and times when it falls. Therefore, a ramp-type demand is allowed to provide a realistic view. Inflation is included in the model. To capture the real-life uncertainties and imprecision of the parameters, an intuitionistic fuzzy number is considered. A numerical example shows that a fuzzy model can achieve the maximum profit at all times, unlike deterministic models. Sensitivity analysis demonstrates the model's real-world applicability to the parameter values. Finally, managerial insights are provided to help the decision-maker make an informed decision.

1. Introduction

In inventory management system, classical Economic Order Quantity (EOQ) models are not always able to remove the real complexity by addressing dynamic factors such as product deterioration, amelioration, inflation, environmental concerns, and demand variability. To bridge these gaps, researchers have developed advanced models to increase the practical applicability. Ramp-type demand refers to a demand pattern that increases or decreases gradually over time in a linear manner, similar to the shape of a ramp. Instead of remaining constant or changing abruptly, the demand grows (or falls) at a steady rate within a specific time interval. In many real-world situations, customer demand does not jump suddenly but rises slowly as a product becomes popular, or declines gradually as a product reaches the end of its life cycle. The demand for a ramp-type system can be represented by this system very easily. Sustainability has great importance nowadays because a sustainable model can reduce environmental pollution. A fuzzy model is also significant as it can

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handle the vagueness and uncertainty in environmental data. Therefore, a sustainable fuzzy model was in great need and so this model.

Main purpose of this research is as follows:

i. The classical model assumes the demand rate is constant and limits the practical applicability of the model. Managers often face a ramp type of demand when dealing with seasonal goods. Therefore, this model enhances the applicability of the classical model.
ii. Inflation of money is crucial for any business, and hence, is included in the model.
iii. Amelioration of items can impact the demand for any items, and hence, is considered in the model.
iv. Holding cost, inflation rate and holding cost are represented by intuitionistic fuzzy numbers to add robustness in decision-making.

The manuscript has the following section: Section 2 is about the literature review of this study. Section 3 presents the problem definition, along with detailed notations and assumptions. Section 4 formulates the mathematical model, both the crisp and fuzzy ones. Section 5 contains numerical examples to validate the crisp and fuzzy models. Section 6 examines the validity and robustness of the formulated model. Finally, Section 7 presents the article's conclusions in detail.

2. Literature Review

Numerous studies have been carried out to extend classical inventory models by incorporating realistic assumptions that better reflect modern business environments. A growing area of focus is the incorporation of environmental impacts in the inventory management system. Paul *et al.* [1] formulated an inventory model of green products that accounts for carbon taxation, showing how such regulatory measures affect replenishment decisions and support environmentally sustainable operations. In this direction, Pervin [2] observed an inventory model with backorders and the effect of carbon emissions on achieving sustainability. Middya *et al.* [3] examined the impact of a three-echelon model allowing a carbon cap and trade policy for green products. A vendor-buyer paradigm with sustainability and remanufacturing of returned goods was examined by Pervin *et al.* [4].

Deterioration and amelioration of items are also widely studied. Vandana and Srivastava. [5] proposed a deteriorating inventory with the impact of ameliorating products under trapezoidal demand. Mahata and De [6] examine the ameliorating items and partial trade credit, providing valuable insights into retailer–supplier relationships using an EOQ model. Hwang [7] studied inventory systems in which items improve over time, using a Weibull distribution for amelioration. Deteriorating items, particularly those with Weibull-type deterioration, have been a key topic of research. Covert and Philip [8] were among the first to explore EOQ models under Weibull deterioration. Studies, like Wu [9], Giri *et al.* [10], and Deng *et al.* [11], expanded this by integrating ramp-type of demand with deterioration, typically modelled with the Weibull distribution for its flexibility and realism.

Inflation has a crucial impact on inventory planning. Mahata and Goswami [12] developed a fuzzy inventory framework with the help of a ramp-type demand and inflation. Valliathal and Uthayakumar [13] inspected an EOQ model for ameliorating products, allowing shortages and ramp-type demand.

Fuzzy sets are considered a tool for handling uncertainty in numerical evaluation. Fuzzy sets are used in many different domains, such as technology and innovation, medical diagnostics, supply chain management, mathematical model and decision making. In this study, we consider triangular and trapezoidal fuzzy sets to address uncertainty and vagueness.

More recently, Chauhan *et al.* [14] demonstrated a model for an inventory with inflation, time-dependent holding costs, and demand of ramp-type. Rajoria *et al.* [15] studied decaying items with the help of partial backlogging and ramp demand in an environment of inflation, highlighting the effects of customer backorder behaviour. Further, De *et al.* [16] describe a model of fuzzy EOQ for items with inadequate quality and quantity-based discounts. Gupta *et al.* [17] studied an inventory model with an inflammatory system and a ramp-type demand and deterioration. Manna *et al.* [18] proposed an effective EOQ model incorporating a time-dependent deterioration rate and unit production cost.

Table 1 below provides a detailed comparison of our work with recent works on different parameters.

Table 1
Detailed comparison of our study with the previous literature

Authors	RT	AR	DT	CT	Inflation	FM
Paul *et al.* [1]	-	-	-	-	-	-
Vandana *et al.* [5]	☑	☑	☑	-	☑	-
Mahata and De [6]	-	☑	-	-	-	-
De *et al.* [16]	-	-	-	-	-	☑
Gupta *et al.* [17]	☑	-	-	-	☑	-
Valliathal *et al.* [13]	☑	-	-	-	☑	-
Chauhan *et al.* [14]	☑	-	-	-	☑	-
Rajoria *et al.* [15]	☑	-	-	-	☑	-
Manna *et al.* [18]	☑	-	-	-	-	-
Covert *et al.* [8]	-	-	☑	-	-	-
Giri *et al.* [10]	☑	-	☑	-	-	-
Deng *et al.* [11]	☑	-	-	-	-	-
Wu [9]	☑	-	☑	-	-	-
Mahata *et al.* [12]	☑	-	-	-	-	☑
Hwang [7]	-	☑	-	-	-	-
Jayshree *et al.* [19]	☑	-	☑	-	☑	-
This Paper	☑	☑	☑	☑	☑	☑

where RT = Ramp Type, AR = Amelioration Rate with Weibull distribution, DT = Deterioration Rate with Weibull distribution, CT = Carbon Tax and FM = Fuzzy Models.

From Table 1, it is observed that there is a subsequent gap in the literature, including intuitionistic fuzzy numbers. There are several models that consider ramp-type demand, amelioration rate, deterioration following the Weibull distribution, green investment, the sustainability concept with a carbon tax policy, a trapezoidal fuzzy model, and intuitionistic fuzzy numbers, separately. This is the very first model incorporating all the variables in a single model and finding the sustainable impact when minimizing the total cost of the system.

3. Notation, Assumptions and Problem Definition

3.1 Notation

The notations used in this study are describe as follows:

Notation	:	Description
$I(t)$	:	Inventory level
W	:	Storage Capacity initially (Units)
C_0	:	Unit ordering cost per order
$D(t)$	:	Time depending Ramp type demand (unit/unit of time)
h	:	Unit holding cost
C_a	:	Unit ameliorating cost

C_d	:	Unit deterioration cost
C_s	:	Unit with the shortest cost
C_{ct}	:	Base carbon tax
A	:	Sensitivity of green investment lies between 0 to 1
g	:	Green investment ($/cycle)
C_H	:	Carbon tax is holding
C_D	:	Carbon tax in deterioration
i	:	Inflation rate
T	:	Cycle length
Z	:	Average profit
$\tilde{Z}$	:	Fuzzy profit
$\widetilde{c_0}$	:	Fuzzy ordering cost
$\tilde{h}$	:	Fuzzy holding cost
$\widetilde{C_a}$	:	Fuzzy ameliorating cost
$\widetilde{C_d}$	:	Fuzzy deterioration cost
$\widetilde{C_s}$	:	Fuzzy shortest cost
$\widetilde{C_{ct}}$	:	Fuzzy base carbon tax

3.2 Assumptions

The following are assumed in this study

A. The ramp-type demand rate $D(t)$ is defined as follows

$$\begin{aligned} D(t) &= (a_1 + b_1 t);\ 0 < t \le t^1 \\ &= D;\ \mathrm{t}_1 < t \le t_2 \\ &= (\mathrm{a}_2 - \mathrm{b}_2\, \mathrm{t});\ \mathrm{t}_3 < t \le T \end{aligned} \tag{1}$$

B. The deterioration rate is specified by the Weibull distribution and is defined

$$\theta(\mathrm{t}) = \alpha\beta t^{\beta-1} \tag{2}$$

where $0 < \alpha \ll 1$ is the shape and β be a scale parameter with $\beta > 0$.

C. Amelioration rate follows the Weibull distribution is defined

$$\mathrm{X(t)} = \mathrm{xy} t^{\mathrm{y}-1} \tag{3}$$

where $0 < x \ll 1$ and $y > 0$.

D. $\mathrm{X(t)} > \theta(\mathrm{t})$.

E. Inventory planning period is infinite.

F. Lead time is considered as negligible.

G. The model permits shortages with partial backlogs.

3.3 Problem Definition

In this study, a deteriorating inventory framework is constructed for ameliorating products. This is a retailer model in which it is regarded as a ramp-type demand rate. The time value of money is considered in the paper, keeping in mind the real-life inflationary situation of the market. To achieve sustainability is a motto of the paper and the reduction in carbon emission is also allowed to use this motto. The graphical view of the proposed inventory level is displayed in Figure 1.

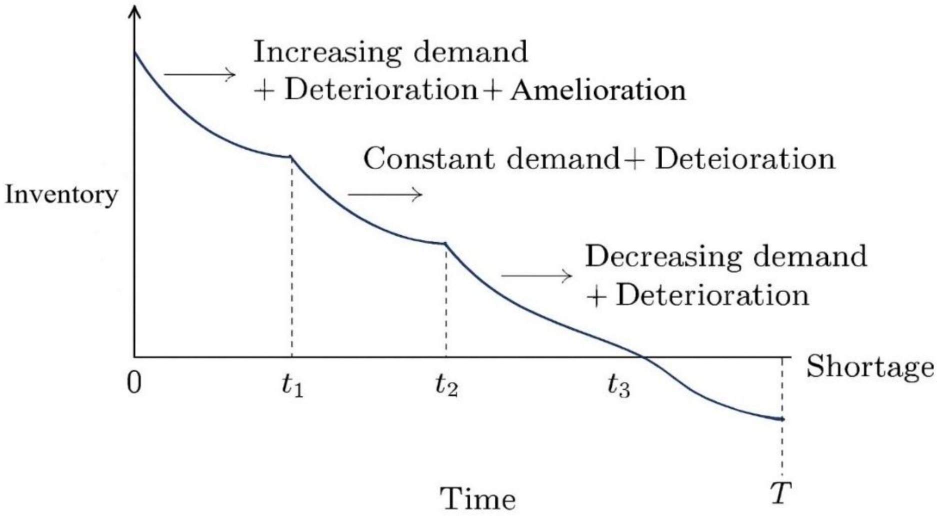

Fig. 1. Graphical structure of the formulated model

4. Mathematical Model

4.1 Crisp Model

Based on Figure 1, the differential equation is formulated as follows:

The differential equation of the inventory level in $[0, t_1]$ is

$$\frac{dI_1(t)}{dt} + \left(\alpha\beta t^{\beta-1} - xyt^{y-1}\right)I(t) = -(a_1 + b_1 t) \tag{4}$$

where $0 \le t \le t_1$, with initial condition $I_1(0) = W$. By solving (1), we get

$$I_1(t) = e^{(xt^y - \alpha t^\beta)}\left[W - a_1\left(t + \frac{\alpha t^{\beta+1}}{\beta+1} - \frac{xt^{y+1}}{y+1}\right) - b_1\left(\frac{t^2}{2} + \frac{\alpha t^{\beta+2}}{\beta+2} - \frac{xt^{y+2}}{y+2}\right)\right] \tag{5}$$

The differential equation of the inventory level in $[t_1, t_2]$ is

$$\frac{dI_2(t)}{dt} + \left(\alpha\beta t^{\beta-1}\right)I(t) = -D \tag{6}$$

where $t_1 \le t \le t_2$. By solving (3), we get

$$I_2(t) = e^{(-\alpha t^\beta)}\left[D\left\{(t_1 - t) + \frac{\alpha\left(t_1{}^{\beta+1} - t^{\beta+1}\right)}{\beta+1}\right\} + e^{xt_1{}^y}\left\{W - a_1\left(t_1 + \frac{\alpha t_1{}^{\beta+1}}{\beta+1} - \frac{xt_1{}^{y+1}}{y+1}\right) - b_1\left(\frac{t_1{}^2}{2} + \frac{\alpha t_1{}^{\beta+2}}{\beta+2} - \frac{xt_1{}^{y+2}}{y+2}\right)\right\}\right] \tag{7}$$

The differential equation of the inventory level in $[t_2, t_3]$ is

$$\frac{dI_3(t)}{dt} + \left(\alpha\beta t^{\beta-1}\right)I(t) = -(a_2 - b_2 t), \tag{8}$$

where $t_2 \le t \le t_3$, with $I(t_3) = 0$. By solving (5), we get

$$I_3(t) = e^{(-\alpha t^\beta)}\left[a_2\left\{(t_3 - t) + \frac{\alpha\left(t_3{}^{\beta+1} - t^{\beta+1}\right)}{\beta+1}\right\} - b_2\left\{\frac{t_3{}^2 - t^2}{2} + \frac{\alpha\left(t_3{}^{\beta+2} - t^{\beta+2}\right)}{\beta+2}\right\}\right] \tag{9}$$

The differential equation of the inventory level in $[t_3, \mathrm{T}]$ is

$$\frac{dI_4(t)}{dt} = -(a_2 - b_2 t), \tag{10}$$

where $t_3 \le t \le T$. By solving (7), we get

$$I_4(t) = -\left[a_2(t - t_3) - b_2\frac{(t^2 - t_3{}^2)}{2}\right]. \tag{11}$$

Since, $I_2(t_2) = I_3(t_2)$, we get

$$W = e^{-xt_1{}^y}\left[a_2\left\{(t_3 - t_2) + \frac{\alpha\left(t_3{}^{\beta+1} - t_2{}^{\beta+1}\right)}{\beta+1}\right\} - b_2\left\{\frac{t_3{}^2 - t_2{}^2}{2} + \frac{\alpha\left(t_3{}^{\beta+2} - t_2{}^{\beta+2}\right)}{\beta+2}\right\}\right.$$

$$+D\left\{(t_2-t_1)+\frac{\alpha(t_2^{\beta+1}-t_1^{\beta+1})}{\beta+1}\right\}\Bigg]+a_1\left(t_1+\frac{\alpha t_1^{\beta+1}}{\beta+1}-\frac{xt_1^{y+1}}{y+1}\right)+\ b_1\left(\frac{t_1^{2}}{2}+\frac{\alpha t_1^{\beta+2}}{\beta+2}-\frac{xt_1^{y+2}}{y+2}\right). \tag{12}$$

4.2 Calculation of cost factors related with this study

1. Ordering cost (OC):

$$OC=c_0e^{-iT}. \tag{13}$$

2. Holding cost (HC):

$$HC=h\int_0^{t_3}I(t)\,e^{-it}\,dt=hH;\ \left(where, H=\int_0^{t_3}I(t)\,e^{-it}\,dt\right)$$

$$=h\int_0^{t_1}e^{(xt^y-\alpha t^\beta-it)}\left[W-a_1\left(t+\frac{\alpha t^{\beta+1}}{\beta+1}-\frac{xt^{y+1}}{y+1}\right)-\ b_1\left(\frac{t^2}{2}+\frac{\alpha t^{\beta+2}}{\beta+2}-\frac{xt^{y+2}}{y+2}\right)\right]dt$$

$$+h\int_{t_1}^{t_2}e^{(-\alpha t^\beta-it)}\Bigg[D\left\{(t_1-t)+\frac{\alpha(t_1^{\beta+1}-t^{\beta+1})}{\beta+1}\right\}$$

$$+\ e^{(xt_1^{y})}\left\{W-a_1\left(t_1+\frac{\alpha t_1^{\beta+1}}{\beta+1}-\frac{xt_1^{y+1}}{y+1}\right)-\ b_1\left(\frac{t_1^2}{2}+\frac{\alpha t_1^{\beta+2}}{\beta+2}-\frac{xt_1^{y+2}}{y+2}\right)\right\}\Bigg]dt$$

$$+h\int_{t_2}^{t_3}e^{(-\alpha t^\beta-it)}\left[a_2\left\{(t_3-t)+\frac{\alpha(t_3^{\beta+1}-t^{\beta+1})}{\beta+1}\right\}-\ b_2\left\{\frac{t_3^{2}-t^2}{2}+\frac{\alpha(t_3^{\beta+2}-t^{\beta+2})}{\beta+2}\right\}\right]dt. \tag{14}$$

3. Amelioration cost (AC):

$$AC=C_a\int_0^{t_1}\mathrm{xy}t^{y-1}I(t)\,e^{-it}\,dt=C_aA;\ \left(where, A=\int_0^{t_1}\mathrm{xy}t^{y-1}I(t)\,e^{-it}\,dt\right)$$

$$=C_a\int_0^{t_1}\mathrm{xy}t^{y-1}\,e^{(xt^y-\alpha t^\beta-it)}\left[W-a_1\left(t+\frac{\alpha t^{\beta+1}}{\beta+1}-\frac{xt^{y+1}}{y+1}\right)-\ b_1\left(\frac{t^2}{2}+\frac{\alpha t^{\beta+2}}{\beta+2}-\frac{xt^{y+2}}{y+2}\right)\right]dt. \tag{15}$$

4. Deterioration cost (DC):

$$DC=C_d\int_0^{t_3}\alpha\beta t^{\beta-1}I(t)\,e^{-it}dt=C_dD, (where\ D=\int_0^{t_3}\alpha\beta t^{\beta-1}I(t)\,e^{-it}dt)$$

$$=C_d\int_0^{t_1}\alpha\beta t^{\beta-1}e^{(xt^y-\alpha t^\beta-it)}\left[W-a_1\left(t+\frac{\alpha t^{\beta+1}}{\beta+1}-\frac{xt^{y+1}}{y+1}\right)-\ b_1\left(\frac{t^2}{2}+\frac{\alpha t^{\beta+2}}{\beta+2}-\frac{xt^{y+2}}{y+2}\right)\right]dt$$

$$+C_d\int_{t_1}^{t_2}\alpha\beta t^{\beta-1}e^{(-\alpha t^\beta-it)}\Bigg[D\left\{(t_1-t)+\frac{\alpha(t_1^{\beta+1}-t^{\beta+1})}{\beta+1}\right\}$$

$$+\ e^{(xt_1^{y})}\left\{W-a_1\left(t_1+\frac{\alpha t_1^{\beta+1}}{\beta+1}-\frac{xt_1^{y+1}}{y+1}\right)-b_1\left(\frac{t_1^2}{2}+\frac{\alpha t_1^{\beta+2}}{\beta+2}-\frac{xt_1^{y+2}}{y+2}\right)\right\}\Bigg]dt$$

$$+C_d\int_{t_2}^{t_3}\alpha\beta t^{\beta-1}e^{(-\alpha t^\beta-it)}\left[a_2\left\{(t_3-t)+\frac{\alpha(t_3^{\beta+1}-t^{\beta+1})}{\beta+1}\right\}-\ b_2\left\{\frac{t_3^{2}-t^2}{2}+\frac{\alpha(t_3^{\beta+2}-t^{\beta+2})}{\beta+2}\right\}\right]dt. \tag{16}$$

5. Shortage cost (SC):

$$SC=-C_s\int_{t_3}^{T}I(t)\,e^{-it}dt=C_sS, \tag{17}$$

where $S=-\int_{t_3}^{T}I(t)\,e^{-it}dt=C_s\int_{t_3}^{T}\left[a_2(t-t_3)-\ b_2\frac{(t^2-t_3^{2})}{2}\right]e^{-it}dt.$

6. Carbon emission cost (CEC):

$$CEC=C_{ct}e^{-Ag}[carbon\ tax\ for\ holding\ inventory$$

$$+carbon\ tax\ for\ deterioration\ of\ inventory]$$

$$=C_{ct}e^{-Ag}\left[C_H\int_0^{t_3}I(t)\,e^{-it}\mathrm{dt}+\ C_D\int_0^{t_3}\alpha\beta t^{\beta-1}I(t)\,e^{-it}\mathrm{dt}\right]=C_{ct}C_E, \tag{18}$$

where $C_E=e^{-Ag}\left[C_H\int_0^{t_3}I(t)\,e^{-it}\mathrm{dt}+\ C_D\int_0^{t_3}\alpha\beta t^{\beta-1}I(t)\,e^{-it}\mathrm{dt}\right].$

Total average cost (Z):

$$Z = \frac{OC + AC + HC + SC + DC + CEC}{T}$$
$$= \frac{c_0 e^{-iT} + hH + C_a A + C_d D + C_s S + C_{ct} C_E}{T}$$
$$= \frac{c_0 e^{-iT}}{T} + \frac{hH + C_a A + C_d D + C_{ct} C_E - \frac{C_s}{i^3}\left[-\left(a_2 - \frac{b_2 t_3}{2}\right)e^{-it_3} i^2 - i\left(\frac{a_2}{t_3} - b_2\right)e^{-it_3} + b_2 \frac{e^{-it_3}}{t_3}\right]}{T}$$
$$+ \frac{C_s}{i^3}\left[-\left(a_2 - \frac{b_2 T}{2}\right)e^{-iT} i^2 - i\left(\frac{a_2}{T} - b_2\right)e^{-iT} + b_2 \frac{e^{-iT}}{T}\right]$$
$$= \frac{c}{T} + \left(c_0 - \frac{C_s a_2}{i^2} + \frac{C_s b_2}{i^3}\right)\frac{e^{-iT}}{T} + \left(-\frac{C_s a_2}{i} + \frac{C_s b_2}{i^2}\right)e^{-iT} + \left(\frac{C_s b_2}{i}\right)T e^{-iT}, \tag{19}$$

where $c = hH + C_a A + C_d D + C_{ct} C_E - \frac{C_s}{i^3}\left[-\left(a_2 - \frac{b_2 t_3}{2}\right)e^{-it_3} i^2 - i\left(\frac{a_2}{t_3} - b_2\right)e^{-it_3} + b_2 \frac{e^{-it_3}}{t_3}\right]$.

To check the optimality condition of Z with respect to T, let us find

$$\frac{dZ}{dT} = -\frac{c}{T^2} + \left(-\frac{c_0 i}{T} + \frac{C_s a_2}{iT} - \frac{C_s b_2}{i^2 T} - \frac{c_0}{T^2} + \frac{C_s a_2}{i^2 T^2} - \frac{C_s b_2}{i^3 T^2} + C_s a_2 - C_s b_2 T\right)e^{-iT}. \tag{20}$$

Putting $\frac{dZ}{dT} = 0$ to find T and putting that value of T in $\frac{d^2 Z}{dT^2}$ and checking that

$$\frac{d^2 Z}{dT^2} = \frac{2c}{T^3} + \left(\frac{2c_o i}{T^2} - \frac{2C_s a_2}{iT^2} + \frac{2C_s b_2}{i^2 T^2} + \frac{2c_0}{T^3} - \frac{2C_s a_2}{i^2 T^3} + \frac{2C_s b_2}{i^3 T^3} - C_s b_2 + \frac{c_o i^2}{T} - \frac{C_s a_2}{T} + \frac{C_s b_2}{iT} - iC_s a_2 + C_s b_2 T i\right)e^{-iT} > 0 \tag{21}$$

which concluded that Z is convex in T.

4.3 Preliminaries of Fuzzy Model

Definition 1. Fuzzy Set:

A fuzzy set (Zadeh [20]) $\tilde{A} \subseteq X$ is defined by

$$\tilde{A} = \{(\mathrm{x}, \mu_{\tilde{A}}(\mathrm{x})) : \mathrm{x} \in \mathrm{X}\} \tag{22}$$

where $\mu_{\tilde{A}} : \mathrm{X} \to [0,1]$ is the membership function (MF) of $\tilde{A}$.

Definition 2. Intuitionistic Fuzzy Set (IFS):

The concept of fuzzy set is generalized by Atanassov [21], defining both the membership and non-membership. The intuitionistic fuzzy set (IFS) is defined as

$$\tilde{A} = \{(\mathrm{x}, \mu_{\tilde{A}}(\mathrm{x}), \nu_{\tilde{A}}(\mathrm{x})) : \mathrm{x} \in \mathrm{X}\} \tag{23}$$

where $\mu_{\tilde{A}}(\mathrm{x}) : \mathrm{X} \to [0,1]$ is the MF and $\nu_{\tilde{A}}(\mathrm{x}) : \mathrm{X} \to [0,1]$ is the non MF satisfying $0 \leqslant \mu_{\tilde{A}}(\mathrm{x}) + \nu_{\tilde{A}}(\mathrm{x}) \leqslant 1$.

Definition 3. Trapezoidal Fuzzy Number (TFN):

A trapezoidal fuzzy number is a fuzzy set $\tilde{A} = (a_1, a_2, a_3, a_4)$, defined on $\mathbb{R}$, where $a_1 < a_2 < a_3 < a_4$ and the MF of $\tilde{A}$ is given by

$$\mu_{\tilde{A}}(\mathrm{x}) = \begin{cases} 0 \ ; \ \mathrm{x} \leqslant a_1 \\ \frac{x - a_1}{a_2 - a_1} \ ; \ a_1 \leqslant \mathrm{x} \leqslant a_2 \\ 1 \ ; \ a_2 \leqslant \mathrm{x} \leqslant a_3 \\ \frac{a_4 - x}{a_4 - a_3} \ ; \ a_3 \leqslant \mathrm{x} \leqslant a_4 \\ 0 \ ; \ \mathrm{x} \geqslant a_4 \end{cases} \tag{24}$$

or,

$$\mu_{\tilde{A}} = \max\left\{\min\left\{\frac{x - a_1}{a_2 - a_1}, 1, \frac{a_4 - x}{a_4 - a_3}\right\}, 0\right\} \tag{25}$$

Definition 4. Defuzzification Formula for TFN:

Chen [22] proposed a defuzzification method for a TFN $\tilde{A} = (a_1, a_2, a_3, a_4)$, using the graded mean integration representation (GMIR) approach and is defined by

$$\text{GMIR}(\tilde{A}) = \frac{\int_0^1 \frac{h}{2}[(a_1+a_4)+h(a_2-a_1-a_4+a_3)]dh}{\int_0^1 hdh}$$
$$= \frac{a_1+2a_2+2a_3+a_4}{6} \quad (26)$$

Definition 5. Triangular Intuitionistic Fuzzy Number (TIFN):

A TIFN (Shaw and Roy [23]) $\tilde{A} \subseteq X$, is defined as $\tilde{A} = (t_1, t_2, t_3)$ on $\mathbb{R}$, where the MF $\left(\mu_{\tilde{A}}(\mathrm{x})\right)$ is defined as

$$\mu_{\tilde{A}}(\mathrm{x}) = \begin{cases} \frac{x-t_1}{t_2-t_1}\ \mu_m\ ;\ t_1 \leqslant \mathrm{x} \leqslant t_2 \\ \frac{t_3-x}{t_3-t_2}\ \mu_m\ ;\ t_2 \leqslant \mathrm{x} \leqslant t_3 \end{cases} \quad (27)$$

and non MF $\left(\nu_{\tilde{A}}(\mathrm{x})\right)$ is defined as

$$\nu_{\tilde{A}}(\mathrm{x}) = \begin{cases} \frac{(t_2-x)+\nu_m(x-t_1)}{t_2-t_1}\ ;\ t_1 \leqslant \mathrm{x} \leqslant t_2 \\ \frac{(x-t_2)+\nu_m(t_3-x)}{t_3-t_2}\ ;\ t_2 \leqslant \mathrm{x} \leqslant t_3 \end{cases} \quad (28)$$

Definition 6. Score function of TIFN:

Consider, $\tilde{A}$ be a TIFN, defined as $\tilde{A} = (t_1, t_2, t_3)$ on $\mathbb{R}$. Then the score function $\left(\mathrm{S}_{\tilde{A}}(\mathrm{x})\right)$ is defined as

$$\mathrm{S}_{\tilde{A}}(\mathrm{x}) = \mu_{\tilde{A}}(\mathrm{x}) - \nu_{\tilde{A}}(\mathrm{x}) \quad (29)$$

Definition 7. α −cut of TIFN:

Consider, $\tilde{A}$ be a TIFN, defined as $\tilde{A} = (t_1, t_2, t_3)$ on $\mathbb{R}$. Then α −cut of $\tilde{A}$ is

$$A(\alpha) = [A_L(\alpha), A_R(\alpha)] = \left[\frac{t_2+t_1(\mu_m-\nu_m)+\alpha(t_2-t_1)}{1+\mu_m-\nu_m}, \frac{t_2+t_3(\mu_m-\nu_m)-\alpha(t_3-t_2)}{1+\mu_m-\nu_m}\right] \quad (30)$$

where $0 \leqslant \alpha \leqslant 1$.

Definition 8. Defuzzification formula for TIFN:

Using α −cut the defuzzification formula (Seikh *et al.* [24]) of a TIFN $\tilde{A} = (t_1, t_2, t_3)$ on $\mathbb{R}$ is given by

$$D = \frac{1}{2}\int_0^1 \left(L^{-1}(\alpha) + R^{-1}(\alpha)\right)d\alpha$$
$$= \frac{1}{2}\left[(t_1 + t_3) + \frac{6t_2-3(t_1+t_3)}{2(1+\mu_m-\nu_m)}\right] \quad (31)$$

4.3 Trapezoidal Fuzzy Model

The crisp inventory model is extended into a fuzzy framework by incorporating trapezoidal fuzzy numbers (TrFN), where the GMIR method is applied for defuzzification (Murthy and Karthigeyan [25]). In this formulation, ordering cost, holding cost, amelioration cost, deterioration cost, storage cost, and carbon emission cost are represented as non-negative TrFN like $\tilde{Y} = (Y_1, Y_2, Y_3, Y_4)$. Accordingly, the average total cost in the fuzzy model is expressed as Z in Equation (1), which is defined by

$$\tilde{Z} = \frac{\tilde{z}_1+2\tilde{z}_2+2\tilde{z}_3+\tilde{z}_4}{6} \quad (32)$$

where $\tilde{z}_k = \frac{\widetilde{c_0}_k e^{-iT} + \tilde{h}_k H + \widetilde{C_a}_k A + \widetilde{C_d}_k D + \widetilde{C_s}_k S + \widetilde{C_{ct}}_k C_E}{T}$ with $k = 1,2,3,4$.

4.4 Intuitionistic Fuzzy Model

In this model, the parameters— ordering cost, holding cost, amelioration cost, deterioration cost, storage cost, and carbon emission cost —are represented as non-negative TIFN of like $\tilde{Y} = (Y_1, Y_2, Y_3)$. By applying the α −cut method to defuzzify Equation (1), we obtain

$$\tilde{Z} = \frac{1}{2}\left[\left(\tilde{Z}_1 + \tilde{Z}_3\right) + \frac{6\tilde{Z}_2 - 3(\tilde{Z}_1 + \tilde{Z}_3)}{2(1+\mu_m - \nu_m)}\right] \tag{33}$$

where $\tilde{Z}_k = \frac{\widetilde{C_0}_k e^{-iT} + \tilde{h}_k H + \widetilde{C_a}_k A + \widetilde{C_d}_k D + \widetilde{C_s}_k S + \widetilde{C_{ct}}_k C_E}{T}$ with $k = 1,2,3,4$.

5. Numerical Examples

Table 2 shows the parametric values of the decision variables and on the basis of which the numerical examples are performed. Figure 2 is the graphical view of the numerical solutions obtained from Mathematica.

Table 2
Parametric values for numerical evaluation

Parameters	Values	Parameters	Values
a_1	200	b_1	12
a_2	330	b_2	23
α	0.6	β	3
x	0.3	y	0.7
W	1000	h	50
C_o	30	C_a	30
C_d	40	C_s	5
C_{ct}	12	A	0.3
g	22	C_H	7
C_D	5	i	0.05
D	200		

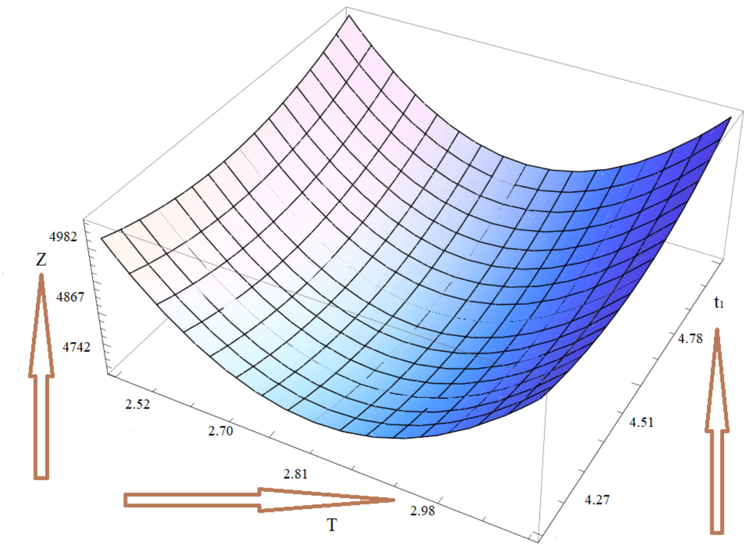

Fig. 2. Graphical representation of Z with respect to t_1 and T

5.1 Crisp model

Solving the model numerically, the values are obtained as $t_1 = 4.11$, $t_2 = 2.14$, $t_3 = 3.46$, $T = 2.61$ and $Z = 4328.40$.

5.2 Trapezoidal Fuzzy Model

Using the same parametric values of all the parameters except $\tilde{C}_0 = (25, 30, 35)$, $\tilde{h} = (45, 50, 55)$, $\tilde{C}_a = (20, 30, 40)$, $\tilde{C}_d = (30, 40, 50)$, $\tilde{C}_s = (3, 5, 7)$ and $\tilde{C}_{ct} = (6, 12, 18)$ in trapezoidal fuzzy model, we get the values as $t_1 = 3.28$, $t_2 = 1.93$, $t_3 = 3.11$, $T = 3.23$ and $\tilde{Z} = 5189.46$.

5.3 Intuitionistic Fuzzy model

Using the same parametric values for all the parameters $\tilde{C}_0 = (15, 25, 30, 45)$, $\tilde{h} = (30, 40, 50, 60)$, $\tilde{C}_a = (10, 20, 30, 40)$, $\tilde{C}_d = (25, 30, 35, 40)$, $\tilde{C}_s = (3, 5, 7, 9)$, $\tilde{C}_{ct} = (7, 12, 17, 22)$ in an intuitionistic fuzzy model, we get the values as $t_1 = 3.21$, $t_2 = 2.54$, $t_3 = 3.98$, $T = 5.71$ and $\tilde{Z} = 5862.29$.

6. Sensitivity Analysis

Changes in parameters a_1, a_2, x, y, C_o, h, C_a, C_d, C_H, g, C_D and i, are analyzed by changing each parameter by $+20\%$,-20%, $+10\%$, -10%. The changes are shown in Table 3 in detail.

Table 3
Results of sensitivity analysis through changes of the value of parameters

Parameter	Percentage change	$t = (t_1, t_2, t_3)$	T	Z	$\tilde{Z}$
a_1	0.72	$(4.35, 2.32, 3.32)$	3.91	4942	---
	0.66		3.86	4817	---
	0.45		3.78	4633	---
	0.48		3.56	4570	---
a_2	3.6	$(4.08, 1.99, 3.06)$	3.45	4712	---
	3.33		3.39	4634	---
	2.7		3.19	4459	---
	2.4		3.07	4307	---
x	0.36	$(3.89, 2.01, 3.14)$	4.45	4829	---
	0.33		4.37	4942	---
	0.27		4.22	5056	---
	0.24		4.18	5123	---
y	0.84	$(4.34, 2.37, 3.50)$	4.76	5166	---
	0.77		4.58	5309	---
	0.63		4.37	5480	---
	0.56		4.10	5542	---
C_o	36	$(4.07, 2.32, 3.21)$	3.97	5980	6422
	33		3.85	5711	6317
	27		3.64	5547	6276
	24		3.55	5376	6148
h	60	$(4.67, 2.54, 3.50)$	3.70	5512	6722
	55		3.64	5432	6523
	45		3.52	5381	6317
	40		3.41	5271	6156
C_a	36	$(4.30, 2.01, 3.35)$	4.62	4989	5632
	33		4.50	4739	5422
	27		4.47	4529	5371
	24		4.39	4321	5208

Table 3
Continued

Parameter	Percentage change	$t = (t_1, t_2, t_3)$	T	Z	$\tilde{Z}$
C_d	48	$(4.21, 2.09, 3.27)$	4.67	5770	6433
	44		4.42	5613	6308
	36		4.20	5540	6229
	32		4.08	5476	6138
C_H	8.4	$(4.00, 2.87, 3.49)$	4.12	5410	5913
	7.7		4.07	5365	5705
	6.3		3.89	5291	5566
	5.6		3.76	5176	5471
g	26.4	$(4.67, 2.56, 3.78)$	4.76	5328	5872
	24.4		4.60	5576	5919
	19.8		4.51	5789	6145
	17.6		4.32	5815	6308
C_D	6	$(4.57, 2.76, 3.69)$	3.99	5732	6534
	5.5		3.60	5612	6310
	4.5		3.33	5502	6275
	4		3.19	5413	6155
i	0.06	$(4.09, 2.03, 3.24)$	4.50	5512	…
	0.55		4.44	5471	…
	0.045		4.36	5238	…
	0.04		4.21	5167	…

It is now noted that an increase in the cost of ordering C_o results in a higher total cost Z and a longer cycle time T. This is because higher setup costs encourage fewer orders, leading to larger order quantities and increased average inventory levels. The total fuzzy cost grows moderately with the rise in C_o, which indicates that the model is under uncertain conditions. As increases of the holding cost h, length of the optimal cycle decreases and increases the total cost significantly. It is observed that the total cost is more sensitive to holding costs rather than the ordering costs due to the effect of inflation. An increase in the amelioration rate x, y leads to a decrease in total cost and an extension in the optimal cycle length. It is observed that improvement in amelioration reduces the effective deterioration and the burden of holding cost, which allows the system to earn more profit. However, when x and y become too large, the marginal benefits diminish due to inflationary influence. Inflation rate i strongly influences both the total cost and cycle length. When the rate of inflation rises, the purchasing power of the system decreases, which leads to higher costs of the system. For ramp-type demand, a higher a_1, a_2 implies a faster-growing demand over time. This increases the total cost and reduces the optimal replenishment cycle length. The model remains stable, but decision-makers should plan shorter cycles to meet rising demand efficiently. An increase in g decreases the total cost of the system, although at first the value of g increases slightly, then the total cost also increases. This system indicates that green investment actually increases the stability and longevity of the system.

7. Managerial Insights

This model helps the inventory manager to take decision in several ways. Inflation in a fuzzy model determines the optimal replenishment policy. Therefore, it enhances the system's reliability by removing uncertainty from the variables. Amelioration rate can impact the holding costs and improve the performance of the decision system. This ameliorating inventory model with a ramp type of demand can assist in controlling holding costs and the rate of inflation. The results imply that moderate green investment is best for optimal benefits and the minimization of total cost.

8. Conclusions and Future Research

This article is about an inventory model that accounts for the time value of money. A Ramp-type of demand over constant demand is assumed in the model. An intuitionistic fuzzy number is used to solve the model for displaying robustness in decision-making.

The model is relatively sensitive to the inflation rate and the holding cost and slightly sensitive to the ordering cost, according to the sensitivity analysis. A fuzzy approach helps to reduce the robustness and uncertainty in the market. The study helps to manage inventories under uncertain, inflationary, and improving product conditions.

This research can be extended in the future by incorporating shortages, preservation technologies, trade credit policies and carbon cap and trade policies for green products.

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Conflicts of Interest
The author declares no conflicts of interest.

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